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OSCILLATIONS OF A TWO-LAYER PLATE UNDER IMPACT LOAD

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Abstract

This article examines the effect of a normal load on an infinite piecewise-homogeneous two-layer plate, when the materials of the upper and lower layers of the plate are elastic. The transverse displacement of points of the contact plane of a two-layer plate is determined, which satisfies the approximate equation obtained in [1], replacing only the viscoelastic operators with the elastic Lamé coefficients, respectively.

Keywords: Oscillation equations, two-layer plate, displacement, elastic, viscoelastic, boundary conditions, initial conditions, operator, Lamé coefficients, differential equation, Fourier integral, complex frequency.

Introduction

The huge scope of construction in our country leads to the need to improve the provisions of building science, which do not adequately meet the increased requirements of building practice. One of these issues is the development of the theory of calculation of multilayer structures (in particular, piecewise homogeneous plates).Plates are one of the main elements of many technical and building structures.In many cases, the plates are not uniform in thickness, in particular, they are piecewise homogeneous (two-layer, etc.) [1,2,3,4,5].

Let us consider the case of forced vibrations of a rectangular two-layer piecewise-homogeneous plate of constant thickness, when both layers of the piecewise-homogeneous two-layer plate are elastic. In real structures, the destruction of their elements is usually accompanied by impact loads.In this work, a solution is constructed for the vibrations of an infinite two-layer plate under the action of a normal load applied to the surface of a two-layer plate[2,3,4,5,6].

The problem is reduced to solving an approximate equation for the transverse displacement W of points in the contact plane of a two-layer plate of constant thickness, obtained in [1,7,8,9,10,11,12].

$$
\begin{aligned}
&Q_1\left(\frac{\partial^4 W}{\partial t^4}\right) + Q_2\left(\Delta\frac{\partial^2 W}{\partial t^4}\right) + Q_3(\Delta^2 W) + Q_4\left(\frac{\partial^6 W}{\partial t^6}\right) + \\
&+Q_5\left(\Delta\frac{\partial^4 W}{\partial t^4}\right) + Q_6\left(\Delta^2\frac{\partial^2 W}{\partial t^2}\right) + Q_7(\Delta^3 W) = F(x,y,t),
\end{aligned}
\qquad (1)
$$

where coefficientsQ_j are determined by the formula obtained in the work [2].

Counting load$F(x,y,t)$ even in *(x, y)*, the transverse displacement W will be sought in the form of Fourier integrals

$$
W = \int_0^\infty \int_0^\infty W_0 \, cos(kx) \, cos(qy) dkdq \qquad (2)
$$

Substituting (2) into equations (1), for W_0 we obtain an ordinary differential equation

$$
W_0^{VI} + A_1 W_0^{IV} + A_2 W_0^{II} + A_3 W_0 = F_0(k,q,t), \qquad (3)
$$

where coefficientsA_j and $F_0(k,q,t)$ are equal:

$$
A_1 = \frac{Q_1' - \gamma^2 Q_5'}{Q_4'}; \;\; A_2 = \frac{\gamma^2(Q_2' - \gamma^2 Q_6')}{Q_4'}; \;\; A_3 = \frac{\gamma^4(Q_3' - \gamma^2 Q_7')}{Q_4'}
$$

$$
F_0(k,q,t) = \int_0^\infty \int_0^\infty F(x,y,t) \, cos(kx) \, cos(qy) dxdy,
$$

and the coefficientsQ_j' are determined by the formulas

$$
\begin{aligned}
&Q_1' = P_2^2(1 + h\rho)^2; \\
&Q_2' = -2P_2^2(2(P_2 D_0 + hD_1)(1 + h\rho) + \\
&\qquad +(P_2 - 1)((1 + h) - (D_0 + hD_1\rho))); \\
&Q_3' = 4(P_2 - 1)(P_2 D_0 + h^2 D_1 + 2hP_2 D_0); \\
&Q_4' = -\frac{1}{6}P_2^2((3h^2\rho^2 + (1 + 4h\rho))(2 - D_0) +
\end{aligned}
\qquad (4)
$$

$$+h^2P_2(3+h\rho(h\rho+4))(2-D_1));$$

$$\begin{gathered}
Q_5' = -\frac{1}{6}P_2(\big(2P_2(4D_0(1-D_0)+1)+(P_2-1)(4-D_0^2)\big)- \\
-P_2h^2\rho^2(2(4D_1^2-4D_1-1)-(P_2-1)D_1(2-D_1))+ \\
+6h^2(\rho(4(P_2^2D_0+D_1)+(P_2-1)(2P_2(1-D_0)-P_2D_1(2-D_0)+ \\
+D_1(1+D_0)))+P_2(1+\rho^2))+\ +2h(2P_2\rho(2+4D_0-D_0^2)- \\
-h^2(2P_2-P_2D_1+5D_1-D_1^2))+(P_2-1)(4-3D_0)+ \\
+2D_1(4-D_0))+2P_2h\rho^2D_0(4-D_1));
\end{gathered} \tag{4}$$

and γ is determined by the formula

$$\gamma^2 = h_0^2(k^2+q),$$

and introduced dimensionless parameters:

$$h=\frac{h_1}{h_0};\ \rho=\frac{\rho_1}{\rho_0};\ b=\frac{b_0}{b_1};\ P_2=\frac{\mu_0}{\mu_1};\ D_0=\frac{1}{2(1-v_0)};\ D_1=\frac{1}{2(1-v_1)}.$$

For ξ, from equation (3) we obtain the frequency equation

$$\xi^6+A_1\xi^4+A_2\xi^2+A_3=0 \tag{5}$$

Frequency equation (5) has purely imaginary roots, i.e. frequencies of natural oscillations. Then, the general solution of the homogeneous differential equation (4) is equal to

$$\begin{gathered}
W_{og} = C_1\,cos(\,\xi_1t)+C_2\,sin(\,\xi_1t)+C_3\,cos(\,\xi_2t)+ \\
+C_4\,sin(\,\xi_2t)+C_5\,cos(\,\xi_3t)+C_6\,sin(\,\xi_3t).
\end{gathered} \tag{6}$$

We write a particular solution of the differential equation (3) in the form

$$W = \frac{1}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)}\{\frac{\xi_2^2-\xi_3^2}{\xi_1}\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_1(t-\zeta)]d\zeta +$$

$$W = \frac{1}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)}\{\frac{\xi_2^2-\xi_3^2}{\xi_1}\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_1(t-\zeta)]d\zeta$$
$$+$$

$$+\frac{\xi_3^2-\xi_1^2}{\xi_2}\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_2(t-\zeta)]d\zeta\,+ \tag{7}$$

$$+\frac{\xi_1^2-\xi_2^2}{\xi_3}\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_3(t-\zeta)]d\zeta\}.$$

Satisfying the zero initial condition, i.e.,

$$W_0=\frac{\partial W_0}{\partial t}=\frac{\partial^2 W_0}{dt^2}=\ldots=\frac{\partial^5 W_0}{\partial t^5}=0, \tag{8}$$

we find that $C_1'=C_2'=\ldots=C_6'=0$. Then, the solution of the problem for the displacement W has the form

$$W=\int_0^\infty\int_0^\infty\frac{cos(\,kx)\,cos(\,qy)}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)}\{\frac{\xi_2^2-\xi_3^2}{\xi_1}\times$$

$$\times\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_1(t-\zeta)]d\zeta\,+ \tag{9}$$

$$+\frac{\xi_3^2-\xi_1^2}{\xi_2}\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_2(t-\zeta)]d\zeta\,+$$

$$+\frac{\xi_1^2-\xi_2^2}{\xi_3}\int_0^t F_0(k,q,\zeta)\,sin[\,\xi_3(t-\zeta)]d\zeta\}dkdq$$

Let if $F(x,y,t)=\sigma_0\delta(x)\delta(y)\delta(z)$,

where σ_0 – constant, voltage dimensions;

$\delta(\zeta)$ – delta is the Dirac function.

Then, the solution of the problem can be written in the form

$$W=\sigma_0\int_0^\infty\int_0^\infty\frac{cos(kx)\,cos(qy)}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)(\xi_3^2-\xi_1^2)}\,[\frac{\xi_2^2-\xi_3^2}{\xi_1}sin(\,\xi_1 t)\,+ \tag{10}$$

$$+\frac{\xi_3^2-\xi_1^2}{\xi_2}sin(\,\xi_2 t)+\frac{\xi_1^2-\xi_2^2}{\xi_3}+sin(\,\xi_3 t)]dkdq$$

Conclusions

From the analytical solution of the problem of the impact of a normal load on the surface of a two-layer plate, it follows that the deflection depends on the geometric and mechanical characteristics of the plate material, and also allows you to accurately describe the stress-strain state of the plate at any point over time.

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