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# HISTORY OF BOUNDARY-TERM PROBLEMS FOR MIXED-TYPE EQUATIONS

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**Abstract:**
Boundary value problems for mixed-type differential equations are of great importance in mathematical physics and engineering calculations. This direction began to take shape in the late 19th and early 20th centuries and developed with the help of classical and modern mathematical analysis methods. At the initial stage of research, scientists such as E. Hopf, A. N. Kolmogorov, J. D. Tamarkin made important contributions. Mixed-type equations are used to describe quasilinear and nonlinear phenomena, which allows for in-depth study of physical processes. Modern research is focused on methods for solving these problems using numerical and analytical methods. This article discusses the historical development of boundary value problems for mixed-type equations and their current significance.

**Keywords**: Mixed-type equations, boundary value problems, mathematics, physics, nonlinear equations, historical development, numerical methods, analytical methods.

## Introduction

Mixed-type equations occupy a special place in mathematical physics and the theory of differential equations. They are used to describe problems encountered in various physical processes, including fluid and gas dynamics, elasticity theory, electrostatics, and quantum mechanics. This article reviews the history of the formation of boundary value problems for mixed-type equations, important studies, and modern approaches.

Many scientists, including A.V. Bitsadze, M.C. Salahiddinov, T.D. Juraev, and their students, have made a great contribution to the study of complex mixed-type

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equations and the creation of methods for their analysis. You can get acquainted with the scientific work done in this direction through the literature.

Mixed-type equations are one of the important areas of modern mathematics. They are inextricably linked with several areas of mathematics, including mathematical analysis, function theory, theory of integral and differential equations, functional analysis, physics, and engineering sciences. Mixed-type equations have been widely developed in recent years. Nowadays, in addition to the classical equations of mathematical physics, mixed-type partial differential equations are also studied, and they are widely used to solve many problems in physics.

## METHODS

The main tasks of studying mixed-type equations are to provide a general understanding of partial differential equations, to identify the types of second-order quasilinear equations and to bring them into canonical form, and to study classical equations and integral equations of mathematical physics, to formulate the main problem for each type of equation, and to study methods for solving these problems. At the same time, the equations of classical mathematical physics, integral equations, and mixed-type equations are studied.

Mixed-type equations are equations that contain algebraic and transcendental (abstract) functions, and to solve them, various algebraic and analytical methods are required.

General form of mixed equations:

$f(x) + g(x) = 0$

Methods for solving such equations:

1. Graphical method – Graphing both functions and finding their intersection points.
2. Approximate method – Determining the corresponding root by checking with approximate values.
3. Newton's method (iterative method) – Iteratively approximating the root using the derivative of the function.
4. Numerical solution methods – Special calculation methods

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**RESULTS**

Mixed-type differential equations include equations of different types (elliptic, hyperbolic, or parabolic) within the same mathematical model. Although such equations initially appeared in classical physics problems, their theoretical foundations began to take shape in the late 19th and early 20th centuries.

Initial studies were conducted by the German mathematician E. Hopf, the Soviet mathematicians A. N. Kolmogorov, S. L. Sobolev, J. D. Tamarkin, and other scientists. These scientists developed methods for finding general solutions for mixed-type equations and presented their physical interpretations.

Boundary-value problems are one of the important research areas for differential equations, which are used to determine how physical processes develop over a certain region or time interval. The boundary conditions for mixed-type equations vary in different regions, making it difficult to solve these problems using classical methods.

At the beginning of the 20th century, significant work was done in mathematical physics on the study of boundary value problems. I. G. Petrovsky, M. V. Keldysh and other scientists studied the issues of correctness, well-definedness and stability of solutions of mixed-type differential equations.[2; 482-b]

The most common boundary value problems for mixed-type equations include the following:

The Dirichlet problem is a case where the boundary values of a function are given.

The Neumann problem is a case where boundary conditions are given through the derivative of a function.

Mixed problems are problems where different types of boundary conditions exist in different areas.

Solving mixed-type equations by traditional methods (analytical approaches) is not always easy. Therefore, in the middle of the 20th century, numerical calculation methods developed rapidly.

Numerical methods: Numerical differentiation and boundary element methods are widely used in solving boundary value problems. With the development of computer technology, numerical modeling and branched algorithms for boundary value problems have been greatly improved.

Analytical methods: Analytical approaches such as Green's functions, Fourier series, integral operators are used for well-posed problems.

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Currently, mixed-type equations and their boundary value problems are used in problems related to artificial intelligence, automated computing methods, and quantum computing. For example:

Gas dynamics and aerodynamics - the study of flows at supersonic speeds.

Medicine and biomechanics - the interaction of fluids and tissues.

Electromagnetic wave theory - the propagation of electromagnetic waves in various materials.[5; 201-b]

**Methods for solving boundary value problems**

Let us consider the main methods for solving boundary value problems for mixed-type nonlinear equations:

1. The method of differences of solutions: This method consists in approximating a differential equation with a system of algebraic equations using network functions. It is widely used in numerical solutions of boundary value problems, but requires significant computational resources for high-dimensional problems.
2. Decision element method: The solution domain is divided into small elements, within which the solution is approximated by polynomials. This method is convenient for solving problems with complex geometries, but requires high accuracy in the choice of mesh and initial conditions.
3. Iteration and Newton methods: Used for approximate solutions of nonlinear equations. These methods require initial approximation and can be sensitive to the choice of this approximation.

Example of solving a boundary value problem

Let us consider an example of a boundary value problem for a mixed-type nonlinear equation. Let the equation be given in the form:

$$u + f(u) = 0$$

where

u is a function depending on two variables x and y, and f(u) is a nonlinear function of u. We assume that the boundary conditions of the first kind are given on the boundaries of the solution domain. The main task is to find the function $u(x, y)$ that satisfies the equation and the boundary conditions.

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**ANALYSIS**

Mixed equations are equations that contain algebraic and transcendental (abstract) functions, and their solution requires the use of various algebraic and analytical methods.

General form of mixed equations:

$$f(x) + g(x) = 0$$

Examples:

1. $x^2 - 2^x = 0$

(Here is an algebraic function, and a is a transcendental function.)

2. $x + \sin x = 0$

(Here is an algebraic function, and a is a trigonometric function.)

3. $\log x = x^2 - 3$

(Here is a logarithmic function, and $x^2 - 3$ is an algebraic function.)[6; 239-b]

Although solving boundary value problems for mixed equations is difficult, they are very important for practical problems. Studying their properties, choosing appropriate methods, and analyzing the stability of the solution is one of the important areas of mathematical physics.

**Methods for solving such equations:**

1. Graphical method - Graphing both functions and finding their intersection points.
2. Approximate method – Finding the corresponding root by checking with approximate values.
3. Newton's method (iterative method) – Iteratively finding the root using the derivative of the function.
4. Numerical solution methods – Special calculation methods (Bisection, Secant, Newton-Raphson, etc.).

The main solution methods for mixed-type equations are as follows:

1. Method of characteristics – Used in hyperbolic regions.
2. Fourier series and integral transformations – Used in elliptic regions.
3. Numerical methods (Boundary element method, Differential methods) – Used for complex boundary conditions.
4. Coordinate transformation method – Useful for converting mixed-type regions into a convenient form.

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Mixed-type equations and their boundary value problems are found in many fields such as aerodynamics, quantum physics, seismic waves, and water dynamics. For example:

The flow around a body moving at a certain speed - in this case, there may be elliptical and hyperbolic regions.

The movement of groundwater - in one region there may be a diffusion process, and in another region there may be a hyperbolic transport process.

## CONCLUSION

In conclusion, mixed-type equations and their boundary value problems are one of the important areas of modern mathematical physics and are used in many areas of engineering and natural sciences. Their development began with classical mathematical research and is reaching a new stage through modern computer modeling technologies. In the future, the combination of numerical and analytical approaches will play an important role in finding more effective solutions to these problems.

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