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PROPAGATION OF WAVES IN NONLINEAR MEDIA

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Abstract

The article presents analytical solutions to problems of propagation of one-dimensional and two-dimensional stationary shock waves in an ideal nonlinearly compressible medium under the influence of intense short-term loads in the form of an explosive pulse.

Keywords: Wave equation, unloading wave, wave front, load profile, shock wave, nonlinearly compressible medium.

Introduction

О РАСПРОСТРАНЕНИИ ВОЛН В НЕЛИНЕЙНОЙ СРЕДЕ

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Аннотация

Приводятся аналитические решения задач о распространении одномерной и двумерной стационарной ударной волны в идеальной нелинейно-сжимаемой среде при воздействии интенсивных кратковременных нагрузок в виде взрывного импульса.

Ключевые слова: волновое уравнение, волна разгрузки, фронт волна, профиль нагрузки, ударная волна, нелинейно-сжимаемая среда**.**

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Annotatsiya

Bir o'lchovli va ikki o'lchovli statsionar zarba to'lqinlarining portlovchi impuls ko'rinishidagi kuchli qisqa muddatli yuklar ta'sirida ideal chiziqli bo'lmagan siqilgan muhitda tarqalishi muammolariga analitik echimlar berilgan.

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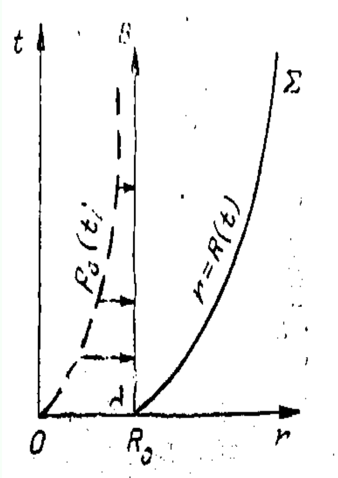
Kalit so'zlar: to'lqin tenglamasi, tushirish to'lqini, to'lqin fronti, yuk profili, zarba to'lqini, chiziqli bo'lmagan siqilgan muhit.

Introduction

Analytical solutions are presented for the propagation of one-dimensional and two-dimensional stationary shock waves in an ideal nonlinearly compressible medium under intense short-term loads in the form of an explosive impulse.



Фиг. 2

One-dimensional nonstationary problems concerning plane and spherical layers are considered, while in the two-dimensional formulation the problem of the action of a moving load on the half-plane of an inelastic medium is solved for the case when the velocity D of the load moving along its boundary exceeds the propagation velocity of the shock wave in the material of the half-plane. In all the problems, it is assumed that the medium at the front is instantaneously loaded in a nonlinear manner, while behind the front, in the disturbed region, linear irreversible loading occurs (Fig. 1). This formulation of the problems makes it possible to solve them by the inverse method, i.e., to prescribe a certain shape (velocity) of the shock-wave front surface and determine the corresponding load profile at the boundary of the layer or half-plane. In this case, the motion of the medium in the unloading region is described by a wave equation in two variables; a Cauchy problem is formulated for it, the solution of which, as is known [1], exists and is unique. A specific example considers the case where the equation of the wave-front surface is specified as a second-degree polynomial with respect to ξ, and the calculation results are compared with those obtained using the method of characteristics [2], which showed satisfactory agreement for all parameters of the medium. Similar problems have been studied by many researchers, in particular N. Mamadaliev, A. Yusupov, B. Donaev, and others.

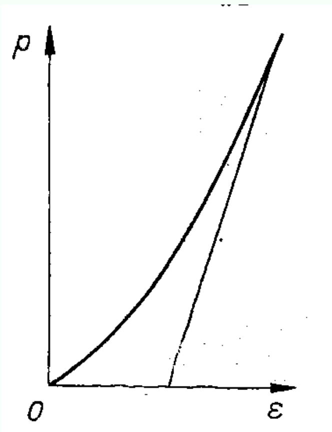


Fig. 1

The case of linear loading and unloading of the medium for a two-dimensional problem is considered in [3, 4]. The solution to the problem of propagation of a converging spherical and cylindrical shock wave in an ideal inelastic medium with rigid unloading is given in [5]. The problems under consideration may have practical applications in studying intense effects in water-saturated soils, as well as in bodies of water.

Problem Statement

Let a monotonically decreasing load $p_0(t)$. be applied at the boundary of the layer. Then a shock wave will propagate in the medium with the front $r = R(t)$, behind which unloading occurs. In this case, for the disturbed region, we have the equations of motion, continuity, and state in the form

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(1.1)

$$\frac{\partial u}{\partial t}+\frac{1}{t}\frac{\partial p}{\partial r}=0,\quad \frac{\partial \rho}{\partial t}+\rho\left(\frac{\partial u}{\partial r}+\frac{\nu u}{r}\right)=0,$$
$$p(r,t)=p^*+E(\varepsilon-\varepsilon^*),$$

where $\varepsilon=1-\frac{\rho_0}{\rho};\quad E=c_p^2\rho.$

At the front $r=R(t)$ we have relations of the form

(1.2) $u^*(t)=\varepsilon^* R,\quad p^*=\rho^*\varepsilon^*\dot{R}^2,\ p^*(t)=\alpha_1\varepsilon^*+\alpha_2\varepsilon^{*2}\quad \left(\dot{R}=\frac{dR}{dt}\right).$

To system (1.1), (1.2), the boundary condition at $r=R_0$ should be added

(1.3) $p=p_0(t).$

Here u –particle velocity; ρ –density; p –pressure; ε –volumetric strain; $\nu=0;2$ correspond to the plane and spherical layers, respectively; the parameters of the medium associated with the front are denoted by a superscript asterisk.

In the case of a plane one-dimensional wave, i.e., for $\nu=0$, the following equation can be obtained from system (1.1)

(1.4) $\frac{\partial^2 u}{\partial t^2}-c_p^2\frac{\partial^2 u}{\partial r^2}=0$,

having the solution of the form

(1.5) $u(r,t)=f_1(r-c_p t)+f_2(r+c_p t),$

where the unknown functions $f_i(\xi)$ $(i=1,2)$ are determined from boundary condition (1.3), taking (1.2) into account. As stated above, the inverse method is used to solve this problem, i.e., the shock-wave propagation law $r=R(t)$. is assumed to be prescribed. Then all parameters of the medium, including $u^*(t)$ and $\varepsilon^*(t)$ at the front Σ c taking (1.2) into account, are known and have the form

(1.6) $\varepsilon^*(t)=\frac{\left(1-\frac{\alpha_1}{\alpha_2}\right)}{2}-\sqrt{\frac{\left(1-\frac{\alpha_1}{\alpha_2}\right)^2}{4}-\frac{\rho_0\dot{R}^2-\alpha_1}{\alpha_2}},$

$$u^*(t)=\dot{R}(t)\varepsilon^*(t).$$

Research results

Thus, for wave equation (1.4) in sector BA**Σ** (Fig. 2), we obtain a modified Cauchy problem with the prescribed parameters (1.6) on curve A**Σ**, in which the first equality, as it were, replaces the condition for the velocity gradient $u^*(t)$. Then, from (1.6), taking (1.1) and (1.5) into account, to determine f_1' и f_2' the following formulas are obtained

(1.7) $f_1'[R(t)-c_p t]=-\frac{\ddot{R}}{2c_p}\left[\Delta_1(t)+\frac{\frac{\rho_0}{\alpha_2}c_p\dot{R}}{\Delta_2(t)}\right],$

$f_2'[R(t)+c_p t]=\frac{\ddot{R}}{2c_p}\left[\Delta_1(t)-\frac{\frac{\rho_0}{\alpha_2}c_p\dot{R}}{\Delta_2(t)}\right],$

where

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$$\Delta_1(t) = \frac{\left(1-\frac{\alpha_1}{\alpha_2}\right)}{2} - \Delta_2(t);$$

$$\Delta_2(t) = \sqrt{\frac{\left(1-\frac{\alpha_1}{\alpha_2}\right)^2}{4} - \frac{\rho_0\dot{R}^2-\alpha_1}{\alpha_2}}.$$

Now, taking (1.7) into account, substituting (1.5) into the first equation of (1.1), and integrating with respect to r from $r = R_0$ to $r = R(t)$ to determine the load $p_0(t)$, we obtain

$$p_0(t) = p^*(t) + \frac{p_0}{2}\left\{\int_{R_0}^{R(t)} \ddot{R}\,[F(z_1)]\left[\Delta_1(F(z_1)) + \frac{\frac{\rho_0}{\alpha_2}c_p\dot{R}(F(z_1))}{\Delta_2(F(z_1))}\right]dr + \int_{R_0}^{R(t)} \ddot{R}\,[F(z_2)]\left[\Delta_1(F(z_2)) - \frac{\frac{\rho_0}{\alpha_2}c_p\dot{R}(F(z_2))}{\Delta_2(F(z_2))}\right]dr\right\}$$

where $z_{1,2} = r \mp c_p t$; $F(z_{1,2}) -$ is the root of the equation $R(t) \mp c_p t = z_{1,2}$ with respect to time t.

In the case of a spherical wave, i.e., for $\nu = 2$, from (1.2) we obtain the equation

$$\frac{\partial^2 u}{\partial t^2} - c_p^2\frac{\partial^2 u}{\partial r^2} - \frac{2c_p^2}{r}\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) = 0,$$

which admits a solution of the form

(1.8) $\quad u(r,t) = \frac{\psi'(r-c_pt)+\Phi'(r+c_pt)}{r} - \frac{\psi(r-c_pt)+\Phi(r+c_pt)}{r^2},$

where the prime denotes differentiation with respect to the argument.

After some transformations, from (1.6), taking (1.8) into account, to determine $\psi''(z_1)$ and $\Phi''(z_2)$, we obtain

$$\psi''(z_1) = \psi''(R_0) + \int_{R_0}^{z_1}\Phi(\xi_1)d\xi_1,$$

(1.9) $\quad \Phi''(z_2) = -\psi''[R(F(z_2)) - c_pF(z_2)] + 2\dot{R}(F(z_2))\Delta_1(z_2) - \frac{\rho_0}{\alpha_2}\frac{\dot{R}(F(z_2))\ddot{R}(F(z_2))}{\Delta_2(z_2)},$

where

$$\Phi(z_1) = -\frac{\dot{R}^3(F(z_1))\Delta_1(F(z_1))}{2c_pR(R-\dot{c}_p)}\left[1 + \frac{R\ddot{R}}{\dot{R}^2}\left(6 + \frac{R\dddot{R}}{\dot{R}\ddot{R}}\right)\right] - \frac{\frac{\rho_0}{\alpha_2}R\dot{R}\ddot{R}^2}{2c_p(\dot{R}-c_p)\Delta_2(F(z_1))}\left[2 + \frac{R\ddot{R}}{\dot{R}^2} - \frac{c_p(\dot{R}+c_p)}{\dot{R}^2}\right] - \frac{\frac{\rho_0}{\alpha_2}R\dot{R}\ddot{R}^2}{2c_p(\dot{R}-c_p)\Delta_2^3(F(z_1))}\left\{\Delta_2^2(F(z_1)\left[2 + \frac{(2\dot{R}+c_p)}{\dot{R}}\left(1 + 2\frac{\dot{R}^2}{R\ddot{R}} + \frac{\dot{R}\dddot{R}}{\ddot{R}^2}\right)\right] + \frac{\rho_0}{\alpha_2}R(2\dot{R}+\dot{c}_p)\right\};$$

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$\psi'(R_0), \psi''(R_0), \Phi'(R_0)$ *are determined from conditions (1.2) as* $t \to 0$. The formula for the load $p_0(t)$ in the case of a spherical wave, taking (1.9) into account, is obtained similarly to the case of a plane wave.

Conclusion

The problems considered may have practical applications in studying intense effects in water-saturated soils, as well as in bodies of water.

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