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WAVE PROPAGATION IN A NONLINEARLY COMPRESSIBLE MEDIUM

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Abstract
The paper considers a plane problem of motion with a supersonic speed D of a monotonically decreasing load along the boundary of a half-plane, the material of which is modeled by an ideal medium with a nonlinear and plastic property. The paper solves a plane problem of motion with a supersonic speed D of a monotonically decreasing load along the boundary of a half-plane, the material of which is modeled by an ideal medium with a nonlinear and plastic property, then a shock wave with a curved surface Σ_p will propagate in the half-plane by assumption the medium on it is instantly loaded and unloading occurs behind the front. The analysis of the obtained formulas for velocity and pressure, as well as the calculation results, show that the unloading process can be achieved if the wave front velocity decays with the depth of the half-plane.

Keywords: Nonlinearly compressible medium, shock wave, unloading wave, wave front, load profile, wave equation.

Introduction

РАСПРОСТРАНЕНИИ ВОЛНЫ В НЕЛИНЕЙНО-СЖИМАЕМОЙ СРЕДЕ

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Аннотация
Рассматривается плоская задача о движении со сверхзвуковой скоростью D монотонно убывающей нагрузки по границе полуплоскости, материал которой моделируется идеальной средой, обладающей нелинейным и

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пластическим свойством. Решается плоская задача о движении со сверхзвуковой скоростью D монотонно убывающей нагрузки по границе полуплоскости, материал которой моделируется идеальной средой, обладающей нелинейным и пластическим свойством, тогда в полуплоскости будет распространяться ударная волна с криволинейной поверхностью Σ_p по предположению среда на ней мгновенно нагружается а за фронтом происходит разгрузка. Анализ получаемых формул скорости и давления, а также результаты расчетов показывают, что процесса разгрузки можно достичь, если скорость фронта волны будет затухать с глубиной полуплоскости.
Отметим, что аналогичный обратный метод применен в задаче о волне разгрузки

Ключевые слова: нелинейно-сжимаемая среда**,** ударная волна, волна разгрузки, фронт волна, профиль нагрузки, волновое уравнение.

CHIZIQLI BO'LMAGAN SIQILADIGAN MUHITDA TO'LQINLARNING TARQALISHI

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Annotatsiya

Maqolada materiali chiziqli bo'lmagan va plastik xususiyatga ega bo'lgan ideal muhit bilan modellashtirilgan yarim tekislik chegarasi bo'ylab monoton ravishda kamayib borayotgan yukning tovushdan yuqori tezligi D bilan harakatning tekis muammosi ko'rib chiqiladi. Qog'oz yarim tekislikning chegarasi bo'ylab monoton ravishda kamayib borayotgan yukning tovushdan yuqori tezlikda D bilan tekis harakati masalasini hal qiladi, uning materiali chiziqli bo'lmagan va plastik xususiyatga ega bo'lgan ideal muhit tomonidan modellashtiriladi, keyin egri sirtli zarba to'lqini S_p bo'lgan zarba to'lqini yarim tekislikda tarqaladi va oldingi yuklama orqasida paydo bo'ladi. Tezlik va bosim uchun olingan formulalar tahlili, shuningdek, hisoblash natijalari shuni ko'rsatadiki, agar to'lqin oldingi tezligi yarim tekislikning chuqurligi bilan pasaysa, tushirish jarayoniga erishish mumkin.

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Kalit so'zlar: chiziqli bo'lmagan siqiladigan muhit, zarba to'lqini, tushirish to'lqini, to'lqin fronti, yuk profili, to'lqin tenglamasi.

Introduction

A plane problem is considered in which a monotonically decreasing load moves at the supersonic speed D along the boundary of a half-plane. The material is modeled as an ideal medium with nonlinear and plastic properties. Under these conditions, a shock wave with a curved surface Σ_p propagates through the half-plane. It is assumed that the medium is loaded instantaneously at the wave surface and unloaded behind the front (Fig. 1).

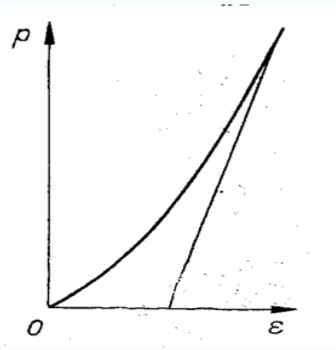

Fig. 1

Problem Statement

In this case, applying the laws of conservation of mass and momentum on the surface Σ_p yields

(1.1) $\rho_0 a = \rho^*(a - v_n^*), \qquad \rho_0 a v_n^* = p^*,$

$$v_\tau^* = 0 \quad (a = D\sin\alpha).$$

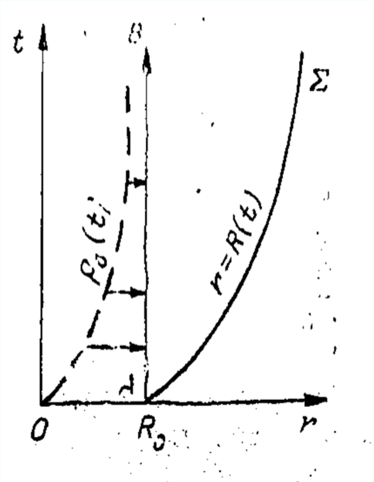

Fig. 2

Since the load profile is assumed to remain unchanged as the wave propagates, the problem is stationary, and in the unloading region, in the moving coordinates $\xi = Dt + x, \ \ \eta = y$, the governing equations are

(1.2) $$D\frac{\partial u}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \xi} = 0, \qquad D\frac{\partial v}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \eta} = 0,$$

$$D\frac{\partial \rho}{\partial \xi} + \rho\left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta}\right) = 0.$$

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The boundary condition has the form

(1.3) for $\eta = 0, \quad \xi \geq 0 \quad p = f(\xi),$

where $f(\xi)$ is a known monotonically decreasing function; v_τ^*, v_n^* are the tangential and normal components of the medium velocity relative to the front Σ_p; u, v are the velocity projections onto the ξ and η axes; α is the angle of inclination of the front Σ_p to the boundary of the half-plane.

To obtain the solution, substitute the first equation of (1.2) into the third equation. Then, for the velocity potential φ, we obtain the wave equation

$$(1.4) \quad \mu^2 \frac{\partial^2\varphi}{\partial\xi^2} - \frac{\partial^2\varphi}{\partial\eta^2} = 0 \qquad \left(\mu^2 = \frac{D^2}{c_p^2} - 1\right),$$

For $D > c_p$, this equation has the solution

$$\varphi(\xi,\eta) = f_3(\xi - \mu\eta) + f_4(\xi + \mu\eta).$$

If a specific form of Σ_p is prescribed, then the velocity components u, v at $\eta = \eta(\xi)$, taking (1.1) into account, can be written as

$$(1.5) \quad u = \frac{\partial\varphi}{\partial\xi} = -D\sin^2\alpha(\xi)\left[\frac{\rho^2D^2}{\alpha_2}\sin^2\alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right],$$

$$v = \frac{\partial\varphi}{\partial\eta} = D\sin\alpha(\xi)\cos\alpha(\xi)\left[\frac{\rho^2D^2}{\alpha_2}\sin^2\alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right],$$

where $\eta(\xi)$ is the equation of the front surface Σ_p. Thus, for a two-dimensional wave inside the curvilinear sector $\xi O \Sigma_p$ (Fig. 2), using (1.4) and (1.5), as in Section 1, we obtain a Cauchy problem, and the functions $f_i(z_i)$ are determined by

(1.6)

$$f_i(z_i) = \mp\frac{D}{\partial\eta}\int_0^{z_i}\frac{tg\alpha[F_i(z_i)]\{1 \pm \mu tg\alpha[F_i(z_i)]\}\Phi_i(z_i)}{\{1 + \mu tg^2\alpha[F_i(z_i)]\}^2}dz_i,$$

where

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$$\Phi_i(z_i) = \left(\frac{\rho_0 D^2}{\alpha_2} - \frac{\alpha_1}{\alpha_2}\right) tg^2\alpha\alpha[F_i(z_i)] - \frac{\alpha_1}{\alpha_2};$$

$F_i(z_i)$ $(i = 3, 4)$ is the root of the equation $\xi \mp \mu\eta(\xi) = z_i$ with respect to ξ; for $i = 3$ in (1.6), the upper sign is used. Note that in the inverse formulation of the problem, i.e., when the shock-wave front surface is specified, condition (1.3) serves as a formula for determining the load profile $f(\xi)$.

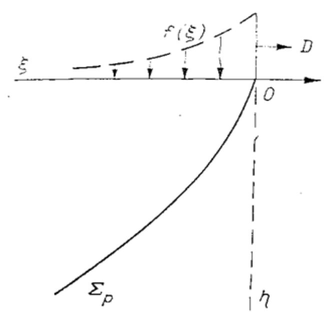

Fig. 3

Thus, taking (1.5) and (1.6) into account, a solution has been obtained for the propagation of a two-dimensional nonlinear wave in a half-plane. Substitution of this solution into (1.3) should yield a decreasing load profile with a sharp front at the origin, while unloading of the medium should occur in the disturbed region.

Analysis of the resulting velocity and pressure formulas, together with the computational results, shows that unloading can be achieved if the wave-front velocity decays with the depth of the half-plane.

It should be noted that a similar inverse method was applied to the unloading-wave problem [6].

To illustrate the method, consider the case in which Σ_p is specified as a second-degree polynomial, i.e.,

(1.7) $$\eta(\xi) = tg\alpha_0\xi - \frac{b}{2}\xi^2.$$

Results of the Study

Calculation results obtained by the analytical method using (1.7) for $tg\alpha_0 = 0{,}1255$, $b = 0{,}86 \cdot 10^{-3}$ and by the method of characteristics [2] are presented in the table, where I denotes the numerical method of characteristics and II denotes the analytical method. Figure 4 shows the variation curves of the pressure and velocity components along the front Σ_p in the half-plane for $b = 0{,}86 \cdot 10^{-3}, b = 0{,}86 \cdot 10^{-2}$ (curves 1 and 2, respectively). The table shows that the results obtained by the two methods are in satisfactory agreement and that the load profile $f(\xi)$ determined by the inverse method decreases monotonically along

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ξ. Figure 4 shows that the pressure p^* and the velocity components u^*, v^* along the front Σ_p decrease linearly with the depth of the half-plane. For $b = 0{,}86 \cdot 10^{-2}$ the decrease in these quantities is more pronounced than for $b = 0{,}86 \cdot 10^{-3}$. Calculations show that all medium parameters, including the pressure at $\eta = 0$ along ξ (on the boundary of the half-plane), decrease differently depending on the coefficient b. For $b = 0{,}86 \cdot 10^{-2}$ the process is more intense and nonlinear. Thus, if the wave-front velocity decays relatively rapidly with depth, the medium parameters, particularly the pressure, also decrease rapidly along the half-plane boundary. However, attenuation of the medium parameters along the boundary $\eta = 0$ occurs more rapidly than at the front.

In general, this work presents an inverse analytical method for solving one- and two-dimensional stationary problems under short-duration intense loading, taking into account nonlinear plastic deformation of an ideal inelastic medium. For $\alpha_2 = 0$, the results of Section 2 coincide with those of [4], obtained using the Mellin transform.

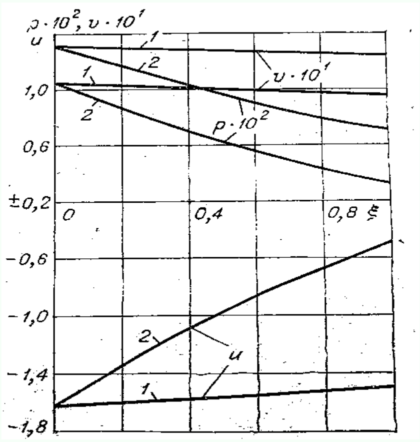

Fig. 4

ξ	u		v		p	
	I	II	I	II	I	II
0	-1,644	-1,644	13,100	13,100	105	105
0,1	-1,628	-1,628	13,025	13,020	103,956	103,937
0,2	-1,610	-1,613	12,944	12,940	102,921	102,979
0,3	-1,597	-1,597	12,861	12,860	101,896	101,958
0,4	-1,581	-1,581	12,780	12,780	100,882	100,937
0,5	-1,565	-1,566	12,699	12,700	99,880	99,978
0,6	-1,550	-1,551	12,621	12,620	98,888	99,021
0,7	-1,535	-1,535	12,543	12,540	97,904	98,000
0,8	-1,519	-1,520	12,466	12,470	96,928	97,042
0,9	-1,505	-1,505	12,390	12,390	95,966	96,085
1,0	-1,490	-1,490	12,314	12,320	95,009	95,127

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Conclusion

The problems considered may have practical applications in studying intense effects in water-saturated soils and bodies of water.

REFERENCES

1. Mikhlin S. G. Course of Mathematical Physics. Moscow: Nauka, 1968.
2. Rakhmatulin Kh. A., Mamadaliyev N. Propagation of Nonlinear Waves in a Soil Half-Space Caused by a Load Moving Along Its Boundary. In: Proceedings of the Symposium "Nonlinear and Thermal Effects in Transient Wave Processes." Tallinn–Gorky, 1973.
3. Skobeev A. M., Flitman L. M. A Moving Load on an Inelastic Half-Plane. PMM, 1970, vol. 34, issue 1, pp. 189–192.
4. Kapustyansky S. M. Propagation and Reflection of Two-Dimensional Plastic Waves. Izvestiya of the USSR Academy of Sciences, Mechanics of Solids, 1973, No. 1, pp. 60–68.
5. Simonov I. V. On a Converging Shock Wave in an Ideally Inelastic Medium and the Stability of Cumulation. PMTF, 1975, No. 5, pp. 82–92.
6. Rakhmatulin Kh. A., Demyanov Yu. A. Strength under Intense Short-Term Loads. Moscow: Fizmatgiz, 1961.
7. Kapustyansky S. M. Propagation and Reflection of Two-Dimensional Plastic Waves. Izvestiya of the USSR Academy of Sciences, Mechanics of Solids, 1973, No. 1.
8. Aidosov A., Mamadaliyev P., Khakimov U. Effect of a Moving Load on a Nonlinearly Compressible Layer with a Rigid Base. PMTF, 1986, No. 3.
9. Rakhmatulin Kh. A., Sagomonyan A. Ya., Alekseev N. A. Problems of Soil Dynamics. Moscow: Moscow State University Press, 1964.
10. Atabayev K. Effect of a Moving Load on a Half-Plane. Reports of the Academy of Sciences of the Uzbek SSR, 1979, No. 9.
11. Rakhmatulin Kh. A., Saatov Ya. U., Sabodash P. F., Filippov I. G. Two-Dimensional Problems of Unsteady Motion of Compressible Media. Tashkent: Fan, 1969.