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THEORETICAL AND PRACTICAL ASPECTS OF FOURIER SERIES AND INTEGRAL TRANSFORMATIONS

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Abstract:
In the current era of digital technologies, signal and image processing is widely used in various fields, including medicine, communications, audio and video technologies, artificial intelligence, and engineering. In these processes, Fourier analysis, that is, Fourier series and integral transforms, is one of the main mathematical methods for performing such important tasks as determining the composition of signals and images, their compression and filtering. This makes it easier to analyze signals and images, clear them of noise, and compress data. This article provides a detailed analysis of the basic concepts of Fourier series and integral transforms, their mathematical foundations, and areas of practical application.

Keywords: Fourier series, Fourier transform, integral transforms, medical imaging and Fourier analysis, biometric identification, artificial intelligence and Fourier analysis.

Introduction

FURYE QATORLARI VA INTEGRAL ALMASHTIRISHLARNING NAZARIY HAMDA AMALIY JIHATLARI

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Annotatsiya
Hozirgi raqamli texnologiyalar davrida signal va tasvirlarni qayta ishlash turli sohalarda, jumladan, tibbiyot, aloqa, audio va video texnologiyalar, sun'iy intellekt hamda muhandislikda keng qo'llaniladi. Ushbu jarayonlarda Furye tahlili, ya'ni Furye qatorlari va integral almashtirishlari, signal va tasvirlarning tarkibini aniqlash, ularni siqish va filtrlash kabi muhim vazifalarni bajarishda asosiy matematik usullardan biri hisoblanadi.Furye qatorlari va integral almashtirishlar yordamida murakkab funksiyalarni sinus va kosinus to'lqinlariga ajratish mumkin. Bu esa signal va tasvirni osonroq tahlil qilish, shovqinlardan tozalash va ma'lumotlarni siqish imkonini beradi. Ushbu maqolada Furye qatorlari va integral almashtirishlarning asosiy tushunchalari, ularning matematik asoslari va amaliy qo'llanilish sohalari haqida batafsil tahlil beriladi.

Kalit so'zlar: Furye qatorlari, Furye transformasi, integral almashtirishlar, tibbiy tasvirlash va Furye tahlili, biometrik identifikatsiya, sun'iy intellekt va Furye tahlili

Fourier series are series that represent periodic functions as an infinite sum of sine and cosine and are based on the following basic formula.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cdot\cos\frac{\pi nx}{L}+b_n\cdot\sin\frac{\pi nx}{L})$$

Here a_n va b_n Fourier coefficients can be calculated as follows.

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cdot\cos\frac{\pi nx}{L}dx\ ;\quad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\cdot\sin\frac{\pi nx}{L}dx$$

L is the half-period of the function.
If the function $f(x)$ is not periodic, it cannot be expanded into a Fourier series. In such cases, the Fourier integral transform is used, meaning the function is represented over an infinite interval (using continuous frequencies instead of integers) and is expressed using the following formula.

$$F(\omega)=\int_{-\infty}^{\infty}f(x)\,e^{-i\omega x}dx\,,$$

Here i - *is a complex number,* ω - – expresses how rapidly the signal changes over time. Accordingly, while the expansion into Fourier series results in discrete

frequencies, the Fourier integral transform produces continuous frequency spectra.
According to the above characteristics, Fourier series and integral transforms are widely used in many areas of our lives, including music and audio processing, electrical engineering, mechanics, telecommunications, medicine, and other fields. Below, the applications in some of these areas are discussed in detail.

Medicine. Fourier series and integral transforms play an important role in medicine, especially in imaging diagnostics and medical signal analysis. Medical imaging technologies such as Magnetic Resonance Imaging (MRI) and Computed Tomography (CT) are based on these mathematical methods. In MRI systems, radio frequency signals are transmitted into the human body and their echoes are recorded. These sets of signals are not suitable for direct processing and are transformed into the frequency domain using the Fourier integral transform. This enables the generation of images based on the electromagnetic properties of different tissues. As a result, physicians can detect diseases or pathologies within the human body.

Engineering and Physics. The Fourier transform is used in engineering and physics for wave modeling and signal processing. In optics, this method is applied to analyze light diffraction and interference, and to reconstruct images through lenses and optical filters. For example, images taken through a telescope or microscope may be distorted due to atmospheric noise or optical errors, but using the Fourier transform, these images can be filtered and restored to a clearer form.

Artificial Intelligence and Digital Image Processing. The collaboration between Fourier series and artificial intelligence enables facial recognition and biometric identification, which are widely used in security systems, banking authentication, and law enforcement for personal identification. The operation of such systems is based on frequency analysis of images and their classification using artificial intelligence. Initially, a human face is scanned using cameras or infrared sensors, and the obtained image is filtered from noise and adjusted for light balance. Typically, the image is represented as a pixel matrix, but direct analysis of such data is complex. Therefore, the Fourier transform is applied to convert the image into the frequency domain.

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As a result, high frequencies in the image represent fine details such as wrinkles, texture, and contours, while low frequencies correspond to major structural elements such as the eyes, nose, lips, and overall face shape. Then, the main components of the facial frequency spectrum are extracted and analyzed by artificial intelligence algorithms. Neural networks compare these features with a pre-existing biometric database. If the face matches a person already in the system, it is confirmed; otherwise, the user may either be registered as a new individual or authentication may be denied.
Using the Fourier transform to extract key facial features not only reduces the data volume but also accelerates the analysis process.

In the field of modern signal processing, Fourier series hold a special place. The expansion of a periodic signal $f(t)$ into a Fourier series is expressed as follows:

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{2\pi nt}{T}\right)+b_n\sin\left(\frac{2\pi nt}{T}\right)\right]$$

The coefficients in this case are defined as follows:

$$a_n=\frac{2}{T}\int_{-T/2}^{T/2}f(t)\cos\left(\frac{2\pi nt}{T}\right)dt,\quad b_n=\frac{2}{T}\int_{-T/2}^{T/2}f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

is calculated by the formulas. Discrete Fourier transforms (DFT) are widely used in practical problems of signal processing, especially in the design of digital filters. The fast algorithm of DFT (FFT) allows to reduce the computational complexity from $O(N^2)$ to O(NlogN), which is important for processing large data sets in real time.
In addition to Fourier integral transforms, Weilet transforms have also gained importance in recent years. Weilet transforms, unlike Fourier transforms, allow for local analysis of signals in both the time and frequency domains. Weilet transforms are expressed as:

$$W_{\psi}f(a,b)=\frac{1}{\sqrt{|a|}}\int_{-\infty}^{\infty}f(t)\psi^*\left(\frac{t-b}{a}\right)dt$$

where ψ(t) is the native wavelet, α is the scaling parameter, and β is the shift parameter. Wavelet permutations play an important role in many practical problems, such as image processing, signal compression, noise reduction, and feature extraction. For example, the JPEG2000 image compression standard is based on wavelet permutations and provides higher performance than JPEG, especially at low bit rates.

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Conclusion

Thus, Fourier series and integral transforms have become one of the methodological foundations of modern science and technology. They are not only of theoretical interest but also of fundamental importance in practical applications. In particular, in medical diagnostics, systems such as Magnetic Resonance Imaging (MRI) and Computed Tomography (CT) are based precisely on Fourier transforms, allowing for the acquisition of highly accurate information about human body tissues. In telecommunications, Fourier analysis forms the basis for efficient signal encoding, multiplexing, and modulation, underpinning modern communication technologies such as Orthogonal Frequency Division Multiplexing (OFDM).

In the field of engineering and technology, Fourier transforms are used for vibration analysis, structural mechanics, and modeling of heat transfer processes. In artificial intelligence and machine learning algorithms, they are widely applied for forming signal feature vectors and improving image and speech recognition systems. In the economy and finance sector, Fourier analysis serves as an effective tool for studying time series, identifying market cycles and trends, and forecasting future events. This significantly enhances the optimization of business processes and the effectiveness of strategic decision-making.

The universality and adaptability of these mathematical methods ensure their application in solving complex problems across many sectors, contributing to resource efficiency, increased productivity, and the development of technological innovations. Modern scientific and technological progress largely relies on the practical implementation of these theoretical foundations.

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