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SOME CONCEPTS OF HOMOMORPHISM AND ISOMORPHISM OF A GROUP

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Abstract:
This article provides an in-depth analysis of the concepts of group homomorphism and isomorphism. A homomorphism is a special map that expresses the relationship between groups and allows algebraic structures to be connected. Isomorphism is defined as the complete equivalence of group structures, proving that they share the same structure. The article provides definitions, properties, and the mathematical significance of these concepts. Furthermore, various examples are given to illustrate their practical applications.

Keywords: Group, homomorphism, isomorphism, algebra, kernel, image, bijectivity, injectivity, surjectivity.

Introduction

GRUPPANING GOMOMORFIZMI VA IZOMORFIZMI HAQIDA BA'ZI BIR TUSHUNCHALAR

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Annotatsiya:
Ushbu maqolada gruppalarning gomomorfizmi va izomorfizmi tushunchalari chuqur tahlil qilinadi. Gomomorfizm gruppalar orasidagi bog'liqlikni

ifodalaydigan maxsus xarita bo‘lib, u algebraik tuzilmalarni bir-biri bilan bog‘lashga imkon beradi. Izomorfizm esa gruppalarning to‘liq ekvivalentligini aniqlab, ularning bir xil strukturaga ega ekanligini isbotlaydi. Maqolada ushbu tushunchalarning ta’riflari, xususiyatlari va ularning amaliy matematikadagi ahamiyati muhokama qilinadi. Bundan tashqari, turli misollar orqali bu tushunchalarning real qo‘llanilishi yoritiladi.

Kalit so’zlar: Gruppa, gomomorfizm, izomorfizm, algebra, yadro, obraz, biyektivlik, inyektivlik, suryektivlik.

Аннотация:
В данной статье подробно анализируются понятия гомоморфизма и изоморфизма групп. Гомоморфизм представляет собой специальное отображение, описывающее связь между группами и позволяющее соединять алгебраические структуры. Изоморфизм, в свою очередь, представляет собой полную эквивалентность между группами и доказывает наличие одинаковой структуры. В статье раскрываются определения, особенности и математическая значимость этих понятий. Кроме того, рассматриваются примеры практического применения данных понятий.

Ключевые слова: Группа, гомоморфизм, изоморфизм, ядро, образ, биективность, инективность, сюрективность.

Introduction
The abstract algebra branch of mathematics requires a deep study of group theory. Groups are a key concept in classifying mathematical structures and studying their properties. To study the relationships between groups, basic concepts such as homomorphism and isomorphism are introduced. Homomorphism is a map connecting groups, which preserves algebraic operations and thus helps to study the relationships between groups. Isomorphism shows that groups have the same structure and proves that the algebraic behavior of these groups corresponds to each other. Homomorphism and isomorphism of groups play an important role in studying their basic properties, forming new groups, and classifying existing groups. This article provides a detailed definition of these concepts, their properties, and examples of their applications.

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Definition: To us$(G;*)$Let (and) be groups. A mapping satisfying the condition for arbitrary elements is called a homomorphism that maps a group G to a group.$G_1;*_1)$ $a, b \in G$ $f(a * b) = f(a) *_1 f(b)$$f: G \to G_1 G_1$

Definition: Let us be given groups and a homomorphism.$GG_1 f: G \to G_1$If f reflectsIf f is surjective and injective, that is, if it is a bijective mapping, then the mapping f is called an isomorphism.

Thus, an isomorphism of groups is a bijective (mutually one-valued) homomorphism mapping the first group to the second group. If there exists an isomorphism mapping a group to a group, then and the groups are said to be mutually isomorphic, defined as$GG_1 GG_1 G \cong G_1$

Definition: and groups, and a homomorphism. Then the set is called the kernel of the homomorphism and is defined as Kerf.$GG_1 f: G \to G_1$ $\{a \in G \mid f(a) = e_1\}f$

Example: Is the following mapping f a homomorphism that maps a group to a group? If so, find its kernel:GG_1

a) $G = (Z, +), G_1 = (Z, +), f(a) = 2a$

For a homomorphism to exist, the following condition must be met:

$f(a + b) = f(a) + f(b)$

Solution: $f(a + b) = 2(a + b) = 2a + 2b = f(a) + f(b)$

Now we will select a number and check it.$\forall\ a, b \in G$

$a = -2, b = 8$

$f\big((-2) + 8\big) = f(-2) + f(8)$

$f(6) = f(-2) + f(8)$

$2 \cdot 6 = 2 \cdot (-2) + 2 \cdot (8)$

$2 \cdot 6 = (-4) + 16$

$12 = 12$

It's a homomorphism. Now we'll find its kernel.

A core is a set of elements that transfer a function to a single element:

$\{a \in G \mid f(a) = e_1\} = Kerf$

$f(a) = 2 \cdot a = 0 \ \cdot \ a = 0$

$Kerf = \{0\}$

The nucleus was found. It is zero.

b) $G = (Z, +),\ \ G_1(Z, +),\ \ f(a) = a + 1$

Solution: Homomorphism condition:$f(a + b) = f(a) + f(b)$

$$f(a + b) = (a + b) + 1 = a + b + 1 \neq f(a) + f(b)$$

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$f(a+b) \neq f(a)+f(b)$The equality is not valid. So our given mapping f is not a homomorphism.

Now let's choose a number for and check it.$\forall\, a, b \in G$

$a = -3, \;\; b = 6$

$f(a+b) = f(a)+f(b)$

$f\big((-3)+6\big)+1 = (f(-3)+1)+(f(6)+1)$

$3+1 = (-2)+7$

$4 \neq 5$

So, this f reflection of ours has no core.

Example: Show that the groups G and G1 are isomorphic.

G= G1$(Z_2, +_2), \; = (\{\pm 1\}, \cdot)$

Solution: – set elements$Z_2\{\bar{0}, \bar{1}\}$We need to set up a match that maps {} to the elements of the collection.± 1

We get the reflection as follows:f: $Z_2 \rightarrow \{\pm 1\}$

$$f(x) = \begin{cases} 1, & \text{agar } x = 0 \text{ bo'lsa} \\ -1, & \text{agar } x = 1 \text{ bo'lsa} \end{cases}$$

Now we check the isomorphism conditions.

a) Homomorphism condition:

$f(a+b) = f(a) \cdot_1 f(b)$

a=0, b=0

$f(\bar{0}+_2\bar{0}) = f(\bar{0}) \cdot f(\bar{0})$

1 = 1 place.

$f(\bar{0}+_2\bar{1}) = f(\bar{0}) \cdot f(\bar{1})$

$(-1) = (-1)$appropriate.

$f(\bar{1}+_2\bar{1}) = f(\bar{1}) \cdot f(\bar{1})$

1 = 1-seater.

Since the combination is 3, we wrote it all down. It satisfies all 3 conditions and it is isomorphic.

$$Z_2 \cong \{\pm 1\}.$$

REFERENCES:

1. Ayupov Sh.A., Omirov B.A. Xudoyberdiyev A.X., Haydarov F.H. Algebra va sonlar nazaryasi. “Toshkent”, 2021 y. 7 bet.
2. Ash R.B. Abstract Algebra. “Dover Publication”, 2006,

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3. Xodjiyev D.X., Faynleyb A.S. Algebra va sonlar nazaryasi kursi. “O‘zbekiston”, 2001 y.
4. Grillet P.A. Abstract Algebra. 2007.
5. Murtozakulov, Z. M. (2023). DIFFERENTIAL OF A GRADED QUASI-FILIFORM LEIBNIS ALGEBRA BY THE NATURAL METHOD. Web of Humanities: Journal of Social Science and Humanitarian Research, 1(9), 20-24.
6. Муртозақулов, З., & Абдуллаев, Ш. (2023). КВАЗИ-ФИЛИФОРМ ЛЕЙБНИЦ АЛГЕБРАЛАРИНИНГ ДИФФЕРЕНЦИАЛИ. Interpretation and researches, 1(1).
7. Murtozaqulov, Z. M. (2023). LOGARIFM VA UNING XOSSALARI. BA’ZI BIR LOGARIFMIK TENGLAMA VA TENGSIZLIKLARNI YECHISH. Academic research in educational sciences, 4(CSPU Conference 1), 409-413.
8. Murtozaqulov, Z. M., & Xambaraliyeva, M. N. (2023). TRIGONOMETRIK TENGLAMALARNI YECHISHNING BA’ZI BIR USULLARI. Academic research in educational sciences, 4(CSPU Conference 1), 404-408.