Educator Insights: A Journal of Teaching Theory and Practice
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6964

TRANSPORTATION PROBLEM: DETERMINING THE OPTIMAL PLAN

Poshaxodjayeva Gulnora Djabborxanovna
Associate Professor of the Department of Higher and Applied Mathematics Tashkent State University of Economics

Abstract:
The transportation problem is one of the most important optimization problems in economics and logistics. This article defines the optimal solution to the transportation problem. The goal is to minimize total transportation costs or maximize prof-its.

Keywords: Transportation costs, point, line, demand, potential.

Introduction

The transportation problem is a specific type of linear programming problem. It can be considered as the problem of finding the optimal transportation plan for shipping goods from supply points to demand points with minimum transportation costs.

According to computational complexity theory, the transportation problem belongs to the class P (polynomial problems). When the total supply does not equal the total demand requested by consumption points, the transportation problem is called unbalanced.

The transportation problem.

Determining the optimal transportation plan (for example, coal) is reduced to solving the following problem:

1. The production capacity A of the i-th production facility (for example, a mine) is given.
2. The demand B of the regions at the j-th consumption point is specified.
3. The cost of transporting 1 ton (for example, of coal) from the i-th production point to the j-th consumption point is given.

Educator Insights: A Journal of Teaching Theory and Practice
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6964

If the number of production points (eg, coal mining locations) is denoted by m, and the number of consumption points by n, the conditions that must be satisfied can be written as:

$$\sum_{j=1}^{n} X\text{ij} = \text{Ai}$$

$$\sum_{i=1}^{m} X\text{ij} = \text{Bj}$$

$$\sum_{i=1}^{m} A\text{i} = \text{j}\sum_{j=1}^{n} B$$

Where

$$\text{Xij} \geq 0 \ .$$

is the volume of transportation from production point i to consumption point j. Condition (1) means that the shipments from production point i must equal its production capacity. Condition (2) means that shipments from different production points to consumer j must satisfy the consumer's demand.

If the total supply does not equal the total demand, the problem can be reformulated by introducing a dummy consumption point whose demand equals the difference between total supply and total demand. The transportation costs to this dummy point are assumed to be zero, meaning the goods directed to the dummy point are not actually transported.

$$Z = \sum_{i,j} c_{ij} x_{ij}$$

subject to conditions (1) and (2).

In this case, the optimization criterion is transportation costs.Sometimes, it is useful to use a different criterion, such as the amount of transportation work measured in ton-kilometers.

Another example of a transportation problem is the optimal distribution of transport vehicles across different routes. Such a distribution can be applied to each mode of transport.

From the base	To the city		
	1	2	3
1	6	4	4
2	3	5	6

Calculate a transportation plan that minimizes transportation costs.

Educator Insights: A Journal of Teaching Theory and Practice
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6964

Let us define xij as the volume of apples transported from the i-base to the j-city. These volumes can be presented in the following table:

From the base	To the city		
	1	2	3
1	X11	X12	X13
2	X21	X22	X23

Since base 1 can provide 300 t capacity,
x11 + x12 + x13 = 3 (hundreds of tons),
and since base 2 can supply 400 tons of apples,
x21 + x22 +x23 = 4 (hundreds of tons).
From the needs of cities it follows that (in hundreds of tons):

x11 + x21 = 3,
x12 + x22 =2,
x13 + x23 = 2.

Total transportation costs are:
Z = 6x11 + 4x12 + 4x13 +3x21 + 5x22 +6x23 .
It is necessary to calculate the unknown xij so that the given system of equations is satisfied and at the same time the objective function z takes on a minimum value.
Let's introduce the following notations:

x11 = x1, x12 = x2, x13 = x3, x21 = x4, x22 = x5, x23 = x6.

Then we get the following system of equations:

x1 + x2 + x3 = 3 (1)
x4 + x5 + x6 = 4 (2)
x1 + x4 = 3 (3)
x2 + x5 = 2 (4)
x3 + x6 = 2 (5)

-6x1 – 4x2 -4x3 -3x4 – 5x5 -6x6 = -z (6)
From equation (3) we obtain:

x4 = 3 – x1,

from equation (4)

x2 = 2 – x5,

from equation (5)

x3 = 2 – x6.

Educator Insights: A Journal of Teaching Theory and Practice
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6964

Substituting these values into equation (1), we obtain:

$$x1 + 2 – x5 + 2 – x6 = 3,$$

$$\text{or } x1 – x5 – x6 = -1,$$

from where $-x1 + x5 + x6 = 1$. (7)
Making the same substitutions in equation (2), we obtain:
$3 – x1 +x5 +x6 = 4$,
Where $-x1 + x5 + x6 = 1$,
i.e., a condition identical to (7).
Substituting x1, x2, x3, x4 into equation (6), we obtain:

$$-6x1 – 4(2 – x5) - 4(2 – x6) -3(3 – x1) – 5x5 – 6x6 = -z,$$

Where $-3x1 – x5 -2x6 = -z + 25$. (8)
From condition (7) we find: $x5 = 1 + x1 – x6$ and substitute this value into equation (8): we get: $-4x1 -x6 = -z + 26$, from where
$Z = 26 + 4x1 + x6$.
The objective function Z takes the minimum value 26 if $x1 = x6 = 0$.
Then $x4 = 3 – x1 =3$, $x3 = 2 – x6 = 2$.
From equation (1) we obtain:
$x1 + x2 + x3 = 3$, or
$0 + x2 + 2 = 3$,
From where $x2 = 1$.
From equation (4) we obtain:
$x5 = 2 – x2 =1$.
Therefore, the optimal transportation plan is:

From the supply base	To the city			Total
	1	2	3	
1 2	x1 = 0 x4 = 3	x2 = 1 x5 = 1	x3 = 2 x6 = 0	3 4
Total	3	2	2	

Minimum transportation costs have been calculated. Let's calculate the cost by substituting the initial data:
Z ten monetary units.$= 4 \times 100 + 4 \times 200 + 3 \times 300 + 5 \times 100 = 2600$

Now let's check the answer using the potential method:
We can fill the table with the numbers specified in the problem statement.

Educator Insights: A Journal of Teaching Theory and Practice
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6964

	300	200	200		
300	6 -4	4 100	4 200		U1 = 0
400	3 300	5 100		-1	U2 = 1
	V1 = 2	V2 = 4	V3 = 4		

Since the number of potentials is five, we will take an arbitrary value for zero (for example, U1 = 0).
The remaining potentials can be found through U1 (Ui + Vj = Cij).

$$V2 + U1 = 4 \quad V2 = 4$$
$$V3 + U1 = 4 \quad V3 = 4$$
$$V2 + U2 = 5 \quad U2 = 1$$
$$V1 + U2 = 3 \quad V1 = 2$$

The initial potentials were determined through the base cells, now we fill the empty cells with these potentials. ij = (Ui + Vj) – CijΔ

$$(2 + 0) -6 = -4<0$$
$$(4 + 1) -6 = -4<0$$

From these calculations it is proven that the selected reference plan is optimal.
Conclusion: The optimal transport plan has been calculated. You can see it in the table and the sequence of equations given above. The purpose of the calculation is to find the minimum costs.

References:

1. H. E. Krynsky. Mathematics for economists // Calculate the optimal transport plan. – P. 303-305, 326-328.
2. Тожибоева, Г. Р., & Умарова, З. (2024). Проблема развития творческих способностей студентов в процессе обучения. Science and innovation, 3(Special Issue 18), 529-532.
3. Rifovna, T. G. (2022). The specificity of the school" rivoge"(development) in the development of the professional competence of the future primary education teacher. Conferencea, 478-480.