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EXAMPLES OF FINDING FUNCTION LIMITS

Zevarjon Rustamboyevna Jumaboyeva
Republic of Uzbekistan
jumaboyevazevarjon9@gmail.com

Abstract:
This article analyzes the domains, formulas, and properties of several important functions in mathematical analysis. For each function, the domain is clearly defined, and some are studied in depth from the perspective of limits. The article serves as a useful methodological guide for students and teachers within the scope of the mathematical analysis course.

Keywords: Function, limit, domain, trigonometric function, exponential function, remarkable limits, mathematical analysis.

Introduction
Functions are one of the fundamental concepts in mathematical analysis. They are widely used in modeling physical, economic, and other real-life processes. In particular, their limits and properties are crucial in solving many applied problems. Among the functions presented below are some remarkable and important ones frequently encountered in mathematical analysis.

Some Fundamental Limit Properties:
Given a $\boldsymbol{X}$ $(\boldsymbol{X} \subset \mathbb{R})$ set let aaa be its limit point, and suppose $\boldsymbol{f(x)}$ va $\mathbf{g}(\boldsymbol{x})$ have finite limits at aaa. Then:

$$\lim_{x\to a} f(x) = b, \quad \lim_{x\to a} g(x) = c$$

1) $\lim\limits_{x\to a}[f(x) \pm g(x)] = b \pm c$

2) $\lim\limits_{x\to a}[f(x) \cdot g(x)] = b \cdot c$

3) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{b}{c}, \quad c \neq 0$

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If two functions are equivalent as $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}=1 \to f(x)\sim g(x)$, then they may be substituted for one another when calculating limits to simplify the computation.

Important and Remarkable Limits:

1. $\lim\limits_{x\to 0}\frac{\sin x}{x}=\lim\limits_{x\to 0}\frac{tgx}{x}=\lim\limits_{x\to 0}\frac{\arcsin x}{x}=\lim\limits_{x\to 0}\frac{arctgx}{x}\lim\limits_{x\to 0}\frac{shx}{x}=\lim\limits_{x\to 0}\frac{thx}{x}=1$

2. $\lim\limits_{n\to\infty}\frac{\sin\alpha x}{x}=\alpha,\alpha\in\mathbb{R}$

3. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=\lim\limits_{x\to\infty}(1+\frac{1}{x})^{x}=e.$

4. $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=1$

5. $\lim\limits_{x\to 0}\frac{a^x-1}{x}=\ln a \quad (a>0).$

6. $\lim\limits_{x\to 0}\frac{e^x-1}{x}=\ln e=1.$

7. $\lim\limits_{x\to 0}\frac{(1+x)^{\alpha}-1}{x}=\alpha \quad (\alpha\in\mathbb{R}).$

8. $\lim\limits_{x\to+\infty}x^0\ln x=\lim\limits_{x\to+\infty}x^{-d}\ln x=\lim\limits_{x\to+\infty}x^c e^{-x}=0 \quad (a>0).$

Proofs of some remarkable and uhum limits:

$\lhd$ **1.2** $\lim\limits_{x\to 0}\frac{tgx}{x}=1$

$$\lim_{x\to 0}\frac{tgx}{x}=\lim_{x\to 0}\frac{\sin x}{x\cos x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}=1\cdot\frac{1}{\cos 0}=1.\rhd$$

$\lhd$ **1.3** $\lim\limits_{x\to 0}\frac{\arcsin x}{x}=1$ $\arcsin x=t$ we calculate by entering a mark: where

$x=\sin t, x\to 0,\Rightarrow t\to 0$

$$\lim_{x\to 0}\frac{\arcsin x}{x}=\lim_{t\to 0}\frac{t}{\sin t}=\lim_{t\to 0}\frac{1}{\sin t}\cdot\frac{t}{t}=\frac{1}{\lim\limits_{t\to 0}\frac{\sin t}{t}}=\frac{1}{1}=1.\ \rhd$$

$\lhd$ **1.4** $\lim\limits_{x\to 0}\frac{\mathrm{arc}\,tgx}{x}=1$ we calculate by entering a mark: where

$arctgx=t$

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$$x = tgt, x \to 0, \Rightarrow t \to 0$$

$$\lim_{x\to 0}\frac{\operatorname{arc}tgx}{x} = \lim_{t\to 0}\frac{t}{tgt} = \lim_{t\to 0}\frac{1}{tgt}\cdot\frac{t}{1} = \frac{1}{\lim\limits_{t\to 0}\frac{tgt}{t}} = \frac{1}{1} = 1. \rhd$$

$\lhd$ **1. 5** $\lim\limits_{x\to 0}\frac{shx}{x} = 1;\quad \lim\limits_{x\to 0}\frac{shx}{x} = \lim\limits_{x\to 0}\frac{e^x - e^{-x}}{2x} = \frac{1}{2}\lim\limits_{x\to 0}\frac{e^x - e^{-x}}{x} = \frac{1}{2}\left(\lim\limits_{x\to 0}\frac{e^x - 1}{x} + \lim\limits_{x\to 0}\frac{e^{-x}-1}{-x}\right) = \frac{1}{2}\cdot 2 = 1.$

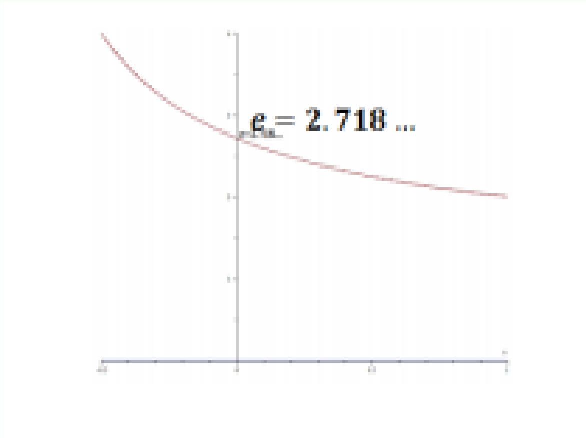
$\rhd$

$\lhd$ **1. 6** $\lim\limits_{x\to 0}\frac{thx}{x} = 1$

$$\lim_{x\to 0}\frac{thx}{x} = \lim_{x\to 0}\frac{\frac{shx}{chx}}{x} = \frac{1}{2}\lim_{x\to 0}\frac{e^x - e^{-x}}{x}\cdot\lim_{x\to 0}\frac{e^x + e^{-x}}{2} = \frac{1}{2}\left(\lim_{x\to 0}\frac{e^x - 1}{x} + \lim_{x\to 0}\frac{e^{-x}-1}{-x}\right)\cdot 1 = \frac{1}{2}\cdot 2 = 1.$$

$\rhd$

2-ajoyib limit: $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \approx 2.71828\ldots$



$$\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{n+1}\cdot\left(1+\frac{1}{n+1}\right)^{-1} = \frac{e}{1} = e$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\cdot\left(1+\frac{1}{n}\right) = e\cdot 1 = e.$$

$$\to\ \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\ .$$

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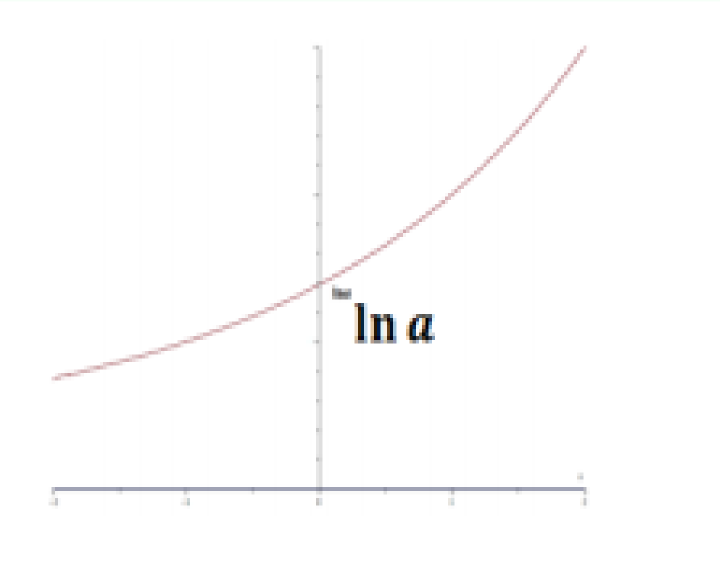
4. $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}=1$

$$\lhd \lim_{x\to 0}\frac{\ln(1+x)}{x}=\lim_{x\to 0}\frac{1}{x}\ln(1+x)=\lim_{x\to 0}\ln((1+x)^{\frac{1}{x}})=\ln\lim_{x\to 0}(1+x)^{\frac{1}{x}}=\ln e=1 \rhd$$

5. $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ln a, \quad a>0.$



$a^x-1=y$, defining it as

$x=\log_a(1+y)$ $x\to 0$ da $y\to 0$

$$\lim_{y\to 0}\frac{y}{\log_a(1+y)}=\lim_{y\to 0}\frac{1}{\dfrac{\log_a(1+y)}{y}}=\lim_{y\to 0}\frac{1}{\log_a((1+y)^{\frac{1}{y}})}=\frac{1}{\log_a\left(\lim\limits_{y\to 0}(1+y)^{\frac{1}{y}}\right)}=\frac{1}{\log_a e}=\ln a.$$

6. $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1.$ we will perform the substitution as above and have the following.

$$\lim_{y\to 0}\frac{y}{\ln(1+y)}=\lim_{y\to 0}\frac{1}{\dfrac{\ln(1+y)}{y}}=\lim_{y\to 0}\frac{1}{\ln((1+y)^{\frac{1}{y}})}=\frac{1}{\ln\left(\lim\limits_{y\to 0}(1+y)^{\frac{1}{y}}\right)}=\frac{1}{\ln e}=1.$$

7. $\lim\limits_{x\to 0}\dfrac{(1+x)^{\alpha}-1}{x}=\alpha \quad (\alpha\in\mathbb{R}).$

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$(1+x)^{\alpha}=e^{t}$ defining it as $x=e^{\frac{t}{\alpha}}-1, x\to 0\Rightarrow t\to 0$

$$\lim_{t\to 0}\frac{e^{t}-1}{e^{\alpha t}-1}=\lim_{t\to 0}\frac{e^{t}-1}{t}\cdot\frac{t}{e^{\alpha t}-1}=\lim_{t\to 0}\frac{e^{t}-1}{t}\cdot\frac{t}{\alpha}\cdot\frac{\alpha}{e^{\alpha t}-1}=\alpha\cdot\lim_{t\to 0}\frac{e^{t}-1}{t}\cdot\frac{1}{\frac{e^{\alpha t}-1}{\alpha t}}=\alpha\cdot\frac{\ln e}{\ln e}=\alpha.$$

8. $\lim\limits_{x\to 0}x^{\alpha}\ln x=\lim\limits_{x\to\infty}x^{-\alpha}\ln x=\lim\limits_{x\to\infty}x^{\alpha}e^{-x}=0,\quad (a>0).$

$\lim\limits_{x\to 0}x^{\alpha}\ln x=0,\ a>0, x^{\alpha}=t,\quad x=\sqrt[\alpha]{t}$

$$x=\sqrt[\alpha]{t}.\ln x=\ln\sqrt[\alpha]{t}=\frac{1}{\alpha}\ln t, x\to 0\quad\Rightarrow\quad t\to 0.$$

$$\lim_{t\to 0}\frac{1}{a}\ln t=\frac{1}{a}\lim_{t\to 0}\ln t=\frac{1}{a}\lim_{t\to 0}\ln(t^{1})=\frac{1}{a}\lim_{t\to 0}\ln\left((1+(t-1))^{t}\right)=$$

$$\frac{1}{a}\lim_{t\to 0}\ln\left((1+(t-1))^{\frac{1}{t-1}}\right)=\frac{1}{a}\lim_{t\to 0}\ln\left(e^{(t-1)t}\right)=\frac{1}{a}\ln e^{0}=\frac{1}{a}\cdot 0=0.$$

Sample examples on the topic:

1-example

$$\lim_{x\to 0}\frac{(1+x)^{4}-1-4x}{x^{2}}$$

Solution: using Newton's binomial formula

$$(1+x)^{4}-1-4x=1+4x+6x^{2}+4x^{3}+x^{4}-1-4x=x^{2}(6+4x+x^{2})$$

$$\lim_{x\to 0}\frac{(1+x)^{4}-1-4x}{x^{2}}=\lim_{x\to 0}\frac{1+4x+6x^{2}+4x^{3}+x^{4}-1-4x}{x^{2}}=\lim_{x\to 0}\frac{x^{2}(6+4x+x^{2})}{x^{2}}$$

$$\lim_{x\to 0}(6+4x+x^{2})=6+0+0=6$$

2-example

$$\lim_{x\to\infty}\left(\sqrt{x^{2}+x+1}-\sqrt{x^{2}-x-1}\right)$$

Solution: when calculating this limit, we save the subtraction from irrationality for which we can extend this subtraction to the subtraction:

$$\lim_{x\to\infty}\frac{\left(\sqrt{x^{2}+x+1}-\sqrt{x^{2}-x-1}\right)\left(\sqrt{x^{2}+x+1}+\sqrt{x^{2}-x-1}\right)}{\sqrt{x^{2}+x+1}+\sqrt{x^{2}-x-1}}=$$

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$$=\lim_{x\to\infty}\frac{x^2+x+1-x^2+x+1}{\sqrt{x^2+x+1}+\sqrt{x^2-x-1}}=\lim_{x\to\infty}\frac{2x+2}{\sqrt{x^2+x+1}+\sqrt{x^2-x-1}}=$$

$$=\lim_{x\to\infty}\frac{2+\frac{2}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}-\frac{1}{x^2}}}=\frac{2+0}{\sqrt{1+0+0}+\sqrt{1-0-0}}=1.$$

3-example

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{x-\frac{\pi}{6}}.$$

Solution: When calculating this limit, we will perform the next replacement $x\to\frac{\pi}{6}$

since $t=x-\frac{\pi}{6}\to 0$, $x=t+\frac{\pi}{6}$

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{x-\frac{\pi}{6}}=\lim_{t\to 0}\frac{2\sin\left(t+\frac{\pi}{6}\right)-1}{t}=\lim_{t\to 0}\frac{2\left(\sin t\cos\frac{\pi}{6}+\cos t\sin\frac{\pi}{6}\right)-1}{t}=$$

$$=\lim_{t\to 0}\frac{2\left(\sin t\cos\frac{\pi}{6}+\sin\frac{\pi}{6}\cos t\right)-1}{t}=\lim_{t\to 0}\frac{\sqrt{3}\sin t+\cos t-1}{t}=\lim_{t\to 0}\frac{\sqrt{3}\sin t+\cos t-1}{t}=$$

$$=\lim_{t\to 0}\frac{2\sqrt{3}\sin\frac{t}{2}\cos\frac{t}{2}-2\sin^2\frac{t}{2}}{t}=\lim_{t\to 0}\frac{\sin\frac{t}{2}}{\frac{t}{2}}\left(\sqrt{3}\cos\frac{t}{2}-\sin\frac{t}{2}\right)=1\cdot(\sqrt{3}-0)=\sqrt{3}.$$

4-example

$$\lim_{x\to 0}\frac{3^x-\sqrt[3]{1+2x}}{\sin 5x+\ln(1-4x)}$$

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$$\lim_{x\to 0}\frac{3^x-\sqrt[3]{1+2x}}{\sin 5x+\ln(1-4x)}=\left[\begin{array}{l} 3^x \sim 1+x\ln 3 \\ \sqrt[3]{1+2x}=(1+2x)^{\frac{1}{3}}\sim 1+\dfrac{2x}{3} \\ \sin 5x\sim 5x \\ 1-4x\sim e^{-4x} \end{array}\right]=$$

$$\lim_{x\to 0}\frac{1+x\ln 3-\left(1-\dfrac{2x}{3}\right)}{5x+\ln e^{-4x}}=\lim_{x\to 0}\frac{x\left(\ln 3+\dfrac{2}{3}\right)}{x}=\ln 3+\frac{2}{3}.$$

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