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ON NONLINEAR PROPAGATION OF A TWO-DIMENSIONAL WAVE

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Abstract
The paper considers a plane problem of the motion of a monotonically decreasing load at supersonic speed D along the boundary of a half-plane, the material of which is modeled by an ideal medium with nonlinear and plastic properties.

Keywords: Plane problem, supersonic speed, ideal medium, half-plane, shock wave, unloading, load profile, stationary and in the unloading region, moving coordinates.

Introduction

We consider a plane problem of motion with a supersonic speed D of a monotonically decreasing load along the boundary of a half-plane, the material of which is modeled by an ideal medium with a nonlinear and plastic property, then a shock wave with a curved surface will propagate in the half-plane, $\Sigma_p$by assumption the medium on it is instantly loaded and unloading occurs behind the front (Fig. 1). In this case, on the surface $\Sigma_p$from the condition of conservation of mass and momentum we obtain

$$(1.1)\quad \rho_0 a = \rho^*(a - v_n^*), \qquad \rho_0 a v_n^* = p^*,$$
$$v_\tau^* = 0 \quad (a = D\sin\alpha).$$

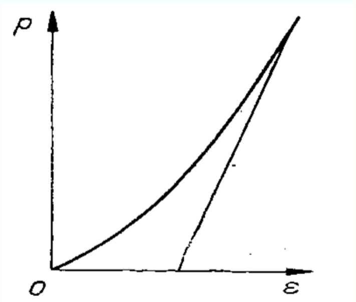

Figure-1

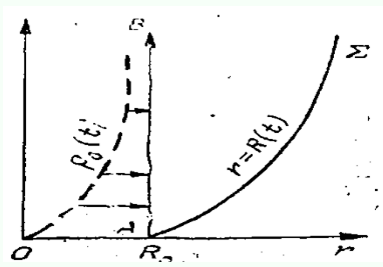

Figure-2

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Since the load profile is considered to be constant as the wave propagates, the problem is stationary and in the unloading region in the moving coordinates $\xi = Dt + x, \quad \eta = y$we have the equations

$$(1.2) D\frac{\partial u}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \xi} = 0, \qquad D\frac{\partial v}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \eta} = 0,$$

$$D\frac{\partial \rho}{\partial \xi} + \rho\left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta}\right) = 0.$$

The boundary condition has the form
(1.3) when$\eta = 0, \quad \xi \geq 0 \quad p = f(\xi)$,
Where $f(\xi)$ −is a known monotonically decreasing function; v_τ^*, v_n^* −tangential and normal components of the velocity of the medium to the front Σ_p; u, v −of the velocity projection on the axes ξ and η; α is the angle of inclination of the front $\Sigma_p$to the boundary of the half-plane [1, 2 , 3].
To obtain a solution to the problem, we substitute the first equation (1.2) into the third. Then φwe obtain the wave equation for the velocity potential

$$(1.4) \mu^2\frac{\partial^2 \varphi}{\partial \xi^2} - \frac{\partial^2 \varphi}{\partial \eta^2} = 0 \qquad \left(\mu^2 = \frac{D^2}{c_p^2} - 1\right),$$

Which is $D > c_p$ has a solution of the form

$$\varphi(\xi, \eta) = f_3(\xi - \mu\eta) + f_4(\xi + \mu\eta).$$

If you set a certain form $\Sigma_p$then the components of the velocity of the medium, u, v при $\eta = \eta(\xi)$taking into account (1.1), are represented in the form

$$(1.5) u = \frac{\partial \varphi}{\partial \xi} = -D\sin^2\alpha(\xi)\left[\frac{\rho^2 D^2}{\alpha_2}\sin^2\alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right],$$

$$v = \frac{\partial \varphi}{\partial \eta} = D\sin\alpha(\xi)\cos\alpha(\xi)\left[\frac{\rho^2 D^2}{\alpha_2}\sin^2\alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right],$$

Where is $\eta(\xi)$ −the equation of the front surface Σ_p. This means that in the case of a two-dimensional wave inside a curvilinear sector $\xi O \Sigma_p$(Fig. 2) for (1.4) taking

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into account (1.5) in the same way as in point 1, we obtain the Cauchy problem and for the definition $f_i(z_i)$we have the formulas

(1 .6)

$$f_i(z_i) = \mp \frac{D}{\partial\eta} \int_0^{z_i} \frac{tg\alpha[F_i(z_i)]\{1 \pm \mu tg\alpha[F_i(z_i)]\}\Phi_i(z_i)}{\{1 + \mu tg^2\alpha[F_i(z_i)]\}^2} dz_i,$$

Where

$$\Phi_i(z_i) = \left(\frac{\rho_0 D^2}{\alpha_2} - \frac{\alpha_1}{\alpha_2}\right) tg^2\alpha\alpha[F_i(z_i)] - \frac{\alpha_1}{\alpha_2};$$

$F_i(z_i)$ $(i = 3, 4)$ —the root of the equation $\xi \mp \mu\eta(\xi) = z_i$with respect to ξ , and in the case $i = 3$ в (1.6)the upper sign is taken. Note that in the inverse formulation of the problem, i.e. for a given surface of the shock wave front, condition (1.3) serves as a formula for determining the load profile$f(\xi)$.

Thus, taking into account (1.5), (1.6), a solution to the problem of propagation of a two-dimensional nonlinear wave in a half-plane is obtained. If we substitute this solution into (1.3), then in principle we should obtain a decreasing load profile with a sharp front at the origin of coordinates, and in the disturbed region the process of unloading the medium should occur [4, 5, 6, 7, 8,].

An analysis of the obtained velocity and pressure formulas, as well as the calculation results, show that the unloading process can be achieved if the wave front velocity decays with the depth of the half-plane [9, 10, 11, 12, 13].

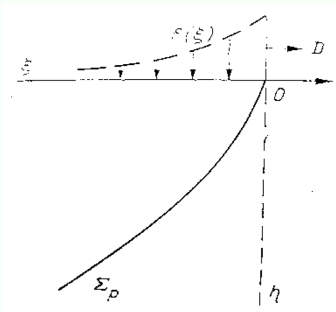

Figure-3

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Note that a similar inverse method is applied in the problem of the unloading wave [6].

As an illustration of the method, we consider the case when Σ_p is given as a second-degree polynomial, i.e.

$$(1.7) \eta(\xi) = tg\alpha_0 \xi - \frac{b}{2}\xi^2.$$

The results of calculations of the analytical method taking into account (1.7) at $tg\alpha_0 = 0{,}1255$, $b = 0{,}86 \cdot 10^{-3}$ and the method of characteristics [1] are presented in the table, where I is the numerical method of characteristics, II is the analytical method. In Fig . 4 the curves of change and velocity along the front of Σ_p the half-plane are given for cases , $b = 0{,}86 \cdot 10^{-3}; 0{,}86 \cdot 10^{-2}$ (curves 1, 2, respectively). From the table it is clear that the results obtained using both methods agree mutually satisfactorily and the load profile found by the inverse method $f(\xi)$ is monotonically decreasing along ξ . From Fig. 4 it is noticeable that the pressure p^* and velocity components u^*, v^* along the front Σ_p decrease with the depth of the half-plane according to a linear law, and in the case $b = 0{,}86 \cdot 10^{-2}$ the decrease in the above values becomes more intense than at , $b = 0{,}86 \cdot 10^{-3}$.Calculations show that all parameters of the medium, including the pressure at $\eta = 0$ along ξ (on the boundary of the half-plane), fall depending on the values of the coefficient b in different ways. In the case , $b = 0{,}86 \cdot 10^{-2}$ this process is more intense and nonlinear [14]. This means that if the wave front velocity attenuates with the depth of the half-plane relatively quickly, then the parameters of the medium, in particular the pressure, along the boundary of the half-plane decrease just as intensively. But the process of attenuation of the parameters of the medium along the boundary $\eta = 0$ occurs faster than at the front [14, 15 , 16, 17, 18, 19].

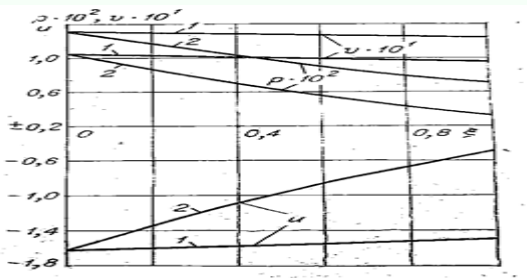

Figure-4

In general, this paper presents an inverse analytical method for solving one-dimensional and two-dimensional stationary problems under short-term intensive impacts, taking into account nonlinear plastic deformation of an ideal inelastic medium. In the case of , $\alpha_2 = 0$the results of section 2 coincide with the results of [4], obtained by applying the Mellin transform .

ξ	u		v		p	
	I	II	I	II	I	II
0	-1,644	-1,644	13,100	13,100	105	105
0,1	-1,628	-1,628	13,025	13,020	103,956	103,937
0.2	-1,610	-1,613	12,944	12,940	102,921	102,979
0.3	-1,597	-1,597	12,861	12,860	101,896	101,958
0.4	-1,581	-1,581	12,780	12,780	100,882	100,937
0.5	-1,565	-1,566	12,699	12,700	99,880	99,978
0.6	-1,550	-1,551	12,621	12,620	98,888	99,021
0.7	-1,535	-1,535	12,543	12,540	97,904	98,000
0.8	-1,519	-1,520	12,466	12,470	96,928	97,042
0.9	-1,505	-1,505	12,390	12,390	95,966	96,085
1.0	-1,490	-1,490	12,314	12,320	95,009	95,127

Conclusion

The problems under consideration may have practical applications in studying intensive impacts in water-saturated soils, as well as in reservoirs.

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