EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

SERIES WITH MULTI-VARIABLE LEVELS

Jumaniyazova Durdona Tuliboy kizi
Lecturer at the Department of Economics at the
Tashkent University of Economics and Technology
G-mail: jdurdona.93@gmail.com

Abstract
This article explores multivariable power series and their properties. The main attention is paid to problems related to the radius of convergence of multidimensional power series, Abel's theorem, Cauchy-Adamard formula, Taylor series expansion, and holomorphic functions. Methods for determining Taylor coefficients in a multivariable case, inequalities for bounded functions, and analytical continuation problems are considered. The study discusses the main theoretical results regarding multidimensional systems of complex analysis.

Keywords: Multivariable power series, radius of convergence, Abel's theorem, Cauchy-Adamard formula, Taylor series, holomorphic function, multidimensional analysis.

Introduction

It is important to study the problems of holomorphic continuation of functions with many complex variables and their applications. This article investigates power series with many variables and their properties. The main attention is paid to problems related to the radius of convergence of multidimensional power series, Abel's theorem, Cauchy-Adamard formula, Taylor series expansion, and holomorphic functions. Methods for determining Taylor coefficients in a multivariable case, inequalities for bounded functions, and analytical continuation problems are considered. The study discusses the main theoretical results regarding multidimensional systems of complex analysis.

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

MAIN PART

Degree rows. Degree rows in the previous paragraph

$$\sum_{n=0}^{\infty} c_n z^n = c_0 + c_1 z + c_2 z^2 + \ldots + c_n z^n + \ldots \quad (c_n \in \square, \quad n = 0,1,2,\ldots)$$

and their radius of convergence, range of convergence, Abel and Cauchy-Adamard theorems, and the Taylor series of the function are studied in detail.

Now let's consider power series with many complex variables and their properties.

1.2.1. This

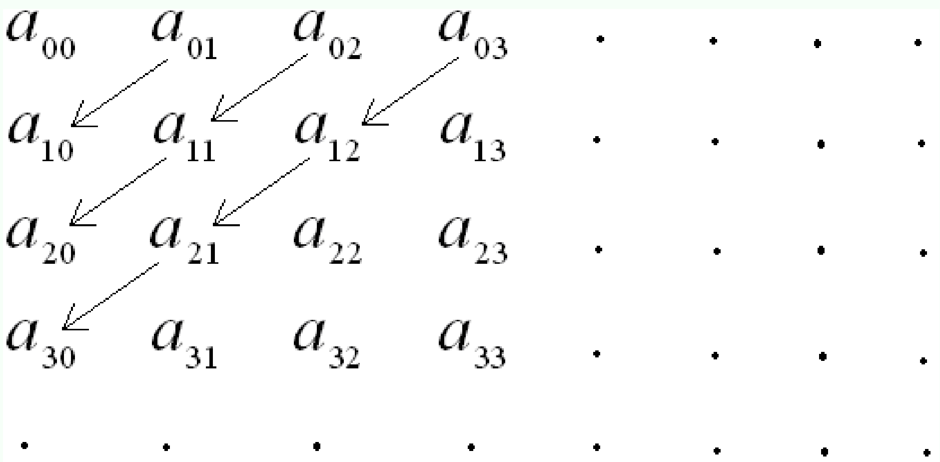
(1.2.1)

An expression is called a level series, where the are indices, and they vary from to . If the indices in (1.2.1) are expressed as follows, in the form of a multi-index, the power series is simplified

$$\sum_{|k|=0}^{\infty} c_k (z-a)^k \quad (|k| = k_1 + k_2 + \ldots + k_n)$$

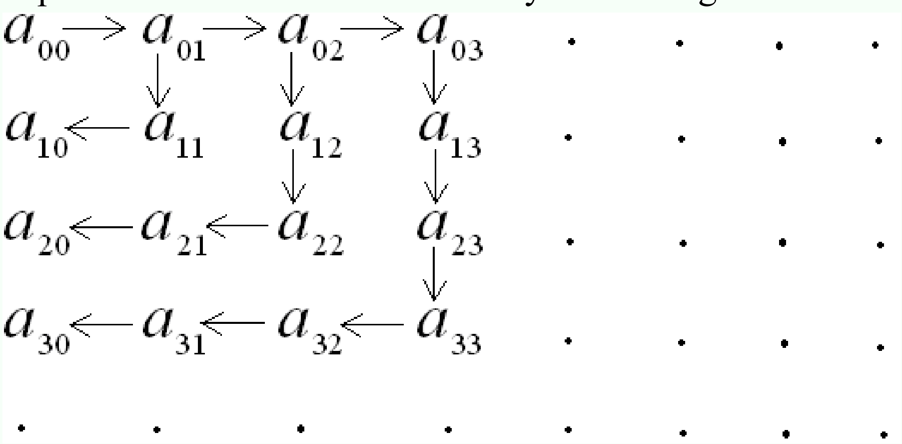
it will be possible to write in the form.

It is known that the sum of a series is defined as the limit of its partial sums. Before introducing the concept of partial sums of the series (1.2.1), it is necessary to sort (number) the multi-index. Let's consider two and multiple indices. If or and these conditions are met, the multi-index is said to be before the multi-index and is denoted as . This sorting is called diagonal sorting, and we use this method.



EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

Multiple indices can also be ordered by the rectangle rule:



By sorting (numbering) the multi-indices, we construct a partial sum of the power series (1.2.1) according to them:

$$F_N = \sum_{j=1}^{N} a_{k(j)} (z-a)^{k(j)}$$

1.2.2-t a r i f. A series of degree (1.2.1) is said to converge at a point, and a series of degree (1.2.2) is said to be its sum.

A is a perfect number. If

$$\sum_{|k|=0}^{\infty} c_k (z-a)^k$$

if the power series converges at a point, then the series

$$U = \{ |z_j - a_j| < |z_j^0 - a_j|, \quad j = 1,2,\dots,n \}$$

is absolutely convergent in a polycircle and uniformly convergent in any compact inside the polycircle.

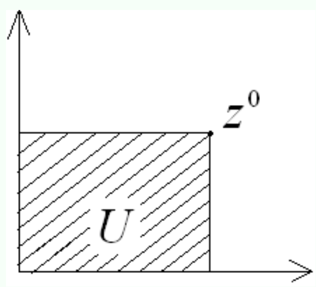


Figure 1.

◄ ketma – ketlikning yaqinlashuvchiligidan, da

$$\left|a_{k(N)}(z^0-a)^{k(N)}\right| = \left|F_N(z^0)-F_{N-1}(z^0)\right| \to 0$$

let's find it. From this

$$a_{k(N)}(z^0-a)^{k(N)}$$

it follows that the sequence is bounded. So,

$$\exists M, \quad \left|a_{k(N)}(z^0-a)^{k(N)}\right| \le M \ , \quad \forall k(N) \ .$$

Obviously,

$$\left|a_k(z-a)^k\right| = \left|a_k(z^0-a)^k\right| \cdot \left|\frac{z-a}{z^0-a}\right|^k$$

At this time, at for an optional multiradius

(1.2.2)

inequality is valid. With ,

.

As is well known,

$$\sum_{|k|=0}^{\infty} Mq^k$$

the numerical series (geometric series) converges. Using this statement and inequality (1.2.2),

$$\sum_{|k|=0}^{\infty} c_k(z-a)^k$$

we find that the power series is absolutely and uniformly convergent in . I'm sorry

Let the set of approximations of the series (1.2.1) be its kernel. is called the approximation region of the series, and it follows from Abel's theorem that is the complete Reinhart region. The maximum radius of a polycircle at some coordinate belonging to is called the bound radius of .

It should be noted that since has a bound radius, an arbitrary

for

condition is met.

Koshi is the Adam's apple formula. The bound radius is this

(1.2.3)

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

satisfies the equality.

<e0> Let there be a bound radius. In that case

$$z_j = a_j + \xi r_j \qquad (j = 1,2,\ldots,n)$$

The complex line is convergent when it is a power series (1.2.1), and divergent when it is a power series (1.2.1). Using the properties of an absolutely convergent series, we find:

$$\sum_{|k|=0}^{\infty} |c_k| (z-a)^k = \sum_{|k|=0}^{\infty} |c_k| r^k \xi^{|k|} = \sum_{j=0}^{\infty} \left(\sum_{|k|=j} |c_k| r^k \right) \xi^j$$

This is a power series according to the Cauchy-Adamar formula

$$\overline{\lim_{j\to\infty}} \sqrt[j]{\sum_{|k|=j} |c_k| r^k} = 1$$

it will be. This equality is equivalent to equality (1.2.3). I'm sorry

If we consider the upper limit in equation (1.2.3), then

$$\Phi(r_1, r_2, \ldots, r_n) = 1$$

which corresponds to the boundary of this surface.

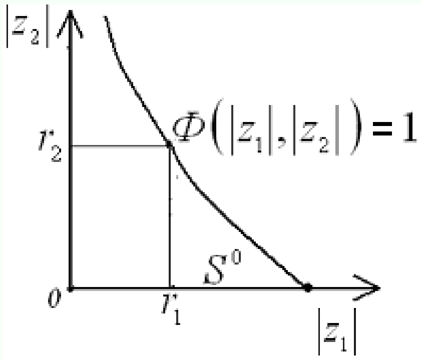


Figure 2.

For example,

$$\sum_{k_1+k_2=0}^{\infty} z_1^{k_1} z_2^{k_2}$$

for row

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

$$\Phi(r_1, r_2) = \overline{\lim_{k_1+k_2\to\infty}} \sqrt[k_1+k_2]{r_1^{k_1} r_2^{k_2}} = \max\{r_1, r_2\}$$

in a series of polycircles.

According to Abel's theorem, a power series converges uniformly within its domain of convergence. Therefore, the sum of the series in the domain is a holomorphic function: . The opposite of this statement is also locally valid.

1.2.1 - theorem. If a function is holomorphic in the domain , it is expanded into a power series around an arbitrary point, i.e., some neighborhood of the point

$$f(z) = \sum_{|k|=0}^{\infty} c_k (z - z^0)^k$$

okay.

Let us consider the polyhedron by assigning the point

Then according to Cauchy's integral formula for a polycircle

(1.2.4)

it will be. For the kernel of this integral, the following (1.2.5.)

we come to equality. Obviously, the series in this equality is uniformly convergent. Consequently, by substituting equality (1.2.5) into (1.2.4), one can swap the positions of the integrals and sums contained therein. Then equality (1.2.4)

(1.2.6)

looks like, here

(1.2.7)

I'm sorry

The series of functions in equality (1.2.7) at the point Taylor series, defined by equality (1.2.7), are called Taylor coefficients.

This means that if a function is holomorphic on some polycircle, it is expanded into a Taylor series on that polycircle, and its coefficients are found using formula (1.2.7).

1.2.2 – Theorem. If a function is bounded on a polycircle, then for the coefficients of the Taylor series of this function

$$|c_k| \le \frac{M}{r_1^{k_1} \dots r_n^{k_n}} \qquad (|k| = 0,1,2,\dots)$$

Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

inequality is valid.

The proof of this theorem is obtained by applying formula (1.2.7) to an arbitrary polycircle, then evaluating and transitioning to the limit at . I'm sorry

References

1. Садуллаев А. Плюрисубгармонические функции // Современные проблемы математики. Fundamental Directions. – M., VINITI, 1985, vol. 8, pp. 65–113. Results of Science and Technology of the VINITI of the USSR Academy of Sciences.
2. Садуллаев А., Чирка Е.М. О продолжении функций с полярными особенностями //Мат.сб. 1987, т. 132, No3, с. 383-390.
3. Sadullaev A.S., Tuychiev T.T. On the extension of Hartog's series that allow holomorphic extension to parallel sections. // Uzb. Math. Journal. 2009, No. 1, p. 148-157.
4. Sadullaev A. Pluripotential Theory. Application. Palmarium academic publishing. Saarbrüchen Deutschland.
5. Tuychiev T.T.. On the analogy of the Hartogz theorem // Izv. AN RUz. Sir, physics. Nauk. 1985, No. 6, pp. 26-29.
6. Tuychiev T.T. Continuation of functions along a fixed direction // Sib. mat. journal. 1988. Vol. 29, No. 3. pp. 142-147.