EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 2, Issue 6, June 2026
brightmindpublishing.com
ISSN (E): 3061-6972

THEORY OF KHARTOGS SERIES AND HOLOMORPHIC FUNCTIONS

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Abstract

This article examines Hartogs series, which play a crucial role in the theory of functions of several complex variables. The problems of analytical continuation of holomorphic functions and their convergence through Hartogs series are considered. The article proves the absolute and uniform convergence of Hartogs series and shows their relationship with plurisubharmonic functions. Also, the importance of Hartogs series in the study of holomorphic functions and their application are explained with examples. The research results offer new approaches that can be applied in the fields of complex analysis and mathematical physics.

Keywords: Hartogs series, holomorphic functions, analytic extension, plurisubharmonic functions, absolute convergence, uniform convergence, functions of complex variables, complex analysis, mathematical physics.

Introduction

In the theory of functions of many complex variables, Hartogs series play an important role alongside Taylor series, Laurent series, and others. Hartogs series are mainly used in the problems of studying holomorphic functions in any direction and their analytical continuation. This article provides an in-depth study of the convergence of Hartogs series, their relationship with plurisubharmonic functions, and their role in the study of holomorphic functions. The absolute and uniform convergence of the Hartogs series is proved, and their significance in complex analysis is shown. The article also discusses the applications of Hartogs series in various fields, including mathematical physics and engineering, and their theoretical and practical significance. The research results contribute to the development of new mathematical models and methods.

MAIN PART

In the theory of functions of many complex variables, along with the Taylor series, the Hartogs series, series by homogeneous polynomials, and Laurent series are also studied. Suppose the function is

$$D = {}'D \times \left\{ |z_n - a_n| < R('z) \right\}$$

let it be holomorphic in the complete Hartogs domain. At each specified d, the function is holomorphic on the domain by its argument. So, this function

(1.3.1).

expands to a power series, where

(1.3.2)

can be. The series in expression (1.3.1) is called the Hartogs series of the function . From equality (1.2.7)

$$c_k('z) = \frac{1}{2\pi i} \int\limits_{|z_n - a_n| = r('z)} \frac{f('z,\xi_n)}{(\xi_n - a_n)^{k+1}}\, d\xi_n = \frac{1}{k!} \frac{\partial^k f('z,z_n)}{\partial z_n^k}\Big|_{z_n = a_n}$$

and from this we see that it will happen. The Hartogs series is used to study holomorphic functions in any direction (here the coordinate axis).

Analytic continuation using Hartogs series. Suppose that $\{z_n = 0\}$ is a Hartogs domain symmetric with respect to the plane

$$D = \{ 'z \in {}'D\ ,\ \ |z_n| < r('z) \} = {}'D \times \{ |z_n| < r('z) \}$$

be given, where $'D \subset C^{n-1}_{'z}$, $0 < r('z) \le \infty$. It is known that if $f(z) \in \mathcal{O}(D)$, the function $f(z)$ is expanded to the Hartogs series:

$$f(z) = \sum_{k=0}^{\infty} c_k('z)\, z_n^k \quad , \quad c_k('z) \in \mathcal{O}('D)\ . \qquad (1.3.3)$$

The radius of convergence of this series is fixed $'z \in {}'D$ at this

$$R('z) = \frac{1}{\overline{\lim\limits_{k\to\infty}} \sqrt[k]{c_k('z)}}$$

is found by the formula. From the following equality, it follows that . So, the function follows

$$u_k('z) = \frac{1}{k}\ \ln|c_k('z)|$$

is the upper limit of the sequence of plurisubharmonic functions.

If

$$R_*('z) = \varliminf_{'w \to 'z} R(w)$$

then the function $R_*('z)$ is half-continuous from below,

$$\hat{D} = 'D \times \{|z_n| < R_*('z)\}$$

the collection will be open.

Theorem 1.3.1. Hartogs Row

$$\sum_{k=0}^{\infty} c_k('z)\ z_n^k$$

Absolute and uniformly convergent within the domain , the sum of which is in the domain

$$F(z) \equiv f(z)$$

it will be.

To prove the theorem, it is sufficient to show the absolute and uniform convergence of the series on any compact . Let's take . Then there will be such that

$$'K \times \{|z_n| \le r_n\} \subset D$$

can be. It is known that $f(z) \in \mathcal{O}(D)$. This function is bounded on the compact under consideration. Including

$$M = \max\{|f(z)|:\ 'z \in 'K\ ,\ |z_n| \le r_n\} < \infty .$$

According to Cauchy inequalities

$$|c_k('z)| \le \frac{M}{r_n^k}\ ,\ 'z \in 'K,\ k = 0,1,2,...,$$

from which we find that , does not depend on the constant . Therefore, the sequence is locally bounded from above in the sequence and

$$[-\ln R('z)]^* = -\ln R_*('z)$$

The function is plurisubharmonic in . At the same time, is a pluripolar set. This means that at almost all points, the radius is equal to .

On the other hand, in the optionally assigned

$$\overline{\lim_{k\to\infty}}\, u_k('z) = -\ln\, R\,('z) \le -\ln R_*\,('z)$$

the relationship is appropriate.

Since , for any region satisfying the condition , there exists a monotonically decreasing sequence such that has . If we enter the notation , then becomes . Then, according to Hartogs's upper limit lemma, for any numbers and , we find such that

$$u_k('z) \le \vartheta_j('z) - \ln(1-\varepsilon) = -\ln(1-\varepsilon) w_j('z)\ , \quad k \ge k_0\ , \quad 'z \in 'K$$

the inequality is satisfied. Using this relationship, we find:

$$|\, c_k('z)\,| \le \frac{1}{[(1-\varepsilon)\ w_j('z)]^k}\ , \quad k \ge k_0, 'z \in K\,.$$

As a result, it is proven that the series (1.3.3) is absolutely and uniformly convergent on the set . At the same time, due to the arbitrariness of the compact , the number , and the fact that has (1.3.3) of the series

$$'D \times \{|z_n| < R_*('z)\}$$

absolute convergence in the domain and uniform convergence within it follows. I'm sorry

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