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PROOF OF SOME INEQUALITIES

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Abstract
This article demonstrates simple methods for proving certain inequalities.

Keywords: Inequality, triangle, Heron's formula, quadratic mean.

Introduction

The future of our country and its prospects primarily depend on the education of youth, their healthy upbringing, and their education in the spirit of national ideals, national ideology, and loyalty to their homeland. It is no secret that this is one of the most urgent tasks. Therefore, today, the education of youth is becoming a priority in the state policy of independent Uzbekistan. Educating young people who can think independently, possess creative abilities, and express their thoughts freely is one of the pressing issues in the educational process. For this purpose, proving inequalities in mathematics education develops qualities such as logical thinking, analytical skills, and creativity.

Inequalities in Triangles

1. Let a, b, c be the sides of a triangle, p - the semi-perimeter, and S - the area of the triangle.

- a) $3(a^2+b^2+c^2) \geq 4p^2$;

- b) $p^2 \geq 3\sqrt{3}$ S.

Proof: Considering that the quadratic mean of three positive numbers is at least as great as their arithmetic mean, we have:

- a) a) $\sqrt{\frac{a^2+b^2+c^2}{3}} \geq \frac{a+b+c}{3} \Rightarrow 3(a^2+b^2+c^2) \geq (a+b+c)^2 = (2p)^2 = 4p^2$

- b) By Heron's formula,

$S^2 = p(p-a)(p-b)(p-c)$. $\frac{S^2}{p} = (p-a)(p-b)(p-c)$,

$\frac{S^2}{p} = (p-a)(p-b)(p-c) \leq \left(\frac{(p-a)+(p-b)+(p-c)}{3}\right)^3 = \left(\frac{3p-(a+b+c)}{3}\right)^3 =$

$\left(\frac{3p-2p}{3}\right)^3 = \frac{p^3}{27}$, т.е. $\frac{S^3}{p} \leq \frac{p^3}{27}$, $S^2 \leq \frac{p^4}{27}$ и $S \leq \frac{p^4}{3\sqrt{3}}$, $p^2 \geq 3\sqrt{3}$ S .

2. Let a, b, c be the sides of a certain triangle, and S the area of the triangle. Prove the inequality:

$a^2+b^2+c^2 \geq 4\sqrt{3}S.$

Proof: Using the fact that the quadratic mean is at least as great as the arithmetic mean, we derive:

$\sqrt{\frac{a^2+b^2+c^2}{3}} \geq \frac{a+b+c}{3}$, $a^2+b^2+c^2 \geq \frac{1}{3}(a+b+c)^2 = \frac{1}{3}(2p)^2 = \frac{4}{3}p^2$

from the previous result.

$a^2+b^2+c^2 \geq \frac{4}{3}p^2 \geq \frac{4}{3} \cdot 3\sqrt{3}\,S = 4\sqrt{3}\,S.$

3. If a, b, c are the sides of a triangle, prove the inequality: $a^2+b^2+c^2 < 2(ab+bc+ca)$

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Proof: By the cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cdot cos\alpha$$
$$b^2 = c^2 + a^2 - 2ca \cdot cos\beta$$
$$c^2 = a^2 + b^2 - 2ab \cdot cos\gamma$$

Summing all three inequalities gives the desired result.

$a^2 + b^2 + c^2 = 2(a^2 + b^2 + c^2) - 2(bc \cdot cos\alpha + ca \cdot cos\beta + ab \cdot cos\gamma),$

$a^2 + b^2 + c^2 = 2(ab\ cos\gamma + bc\ cos\alpha + ca\ cos\beta) < \ 2(ab + bc + ca),$

$cos\gamma < 1,\ cos\alpha < 1,\ cos\beta < 1\ .$

4. If a, b, c are the sides of a triangle, prove the inequality: $a^3 + b^3 + 3abc > c^3$

Proof: Clearly, for the sides of a triangle $a + b > c$, we have:

$a^3 + b^3 + 3abc = (a + b)(a^2 - ab + b^2) + 3abc > c(a^2 - ab + b^2) +$
$+3abc = c(a^2 - ab + b^2 + 3ab) = c(a^2 + 2ab + b^2) = c(a + b)^2 > c \cdot c^2 =$
$= c^3, \quad a^3 + b^3 + 3abc > c^3.$

5. If a, b, c are the sides of a triangle, prove the inequality: $\frac{a^2+2bc}{b^2+c^2} + \frac{b^2+2ac}{c^2+a^2} + \frac{c^2+2ab}{a^2+b^2} > 3.$

Proof: Rearranging gives: $\frac{a^2+2bc}{b^2+c^2} - 1 + \frac{b^2+2ac}{c^2+a^2} - 1 + \frac{c^2+2ab}{a^2+b^2} - 1 > 0$

$\frac{a^2+2bc-b^2-c^2}{b^2+c^2} + \frac{b^2+2ac-c^2-a^2}{c^2+a^2} + \frac{c^2+2ab-a^2-b^2}{a^2+b^2} > 0,$

$\frac{a^2-(b-c)^2}{b^2+c^2} + \frac{b^2-(c-a)^2}{c^2+a^2} + \frac{c^2-(a-b)^2}{a^2+b^2} > 0,$

$\frac{(a-b+c)(a+b-c)}{b^2+c^2} + \frac{(b-c+a)(b+c-a)}{c^2+a^2} + \frac{(c-a+b)(c+a-b)}{a^2+b^2} > 0.$

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If we use the condition that $a+b>c$, $b+c>a$, $c+a>b$ which is correct for an arbitrary triangle, we can observe that every number of fraction is positive, hence the sum is positive.

6. If a, b, c are the sides of a triangle, prove the inequality: $\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}\geq 3$.

Proof:

$$\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}=\frac{a^2}{ab+ac-a^2}+\frac{b^2}{bc+ba-b^2}+\frac{c^2}{ca+cb-c^2}\geq$$

$$\geq\frac{(a+b+c)^2}{2(ab+bc+ca)-(a^2+b^2+c^2)}=\frac{a^2+b^2+c^2+2(ab+bc+ca)}{2(ab+bc+ca)-(a^2+b^2+c^2)}=$$

$$=\frac{2(ab+bc+ca)-(a^2+b^2+c^2)+2(a^2+b^2+c^2)}{2(ab+bc+ca)-(a^2+b^2+c^2)}=1+\frac{2(a^2+b^2+c^2)}{2(ab+bc+ca)-(a^2+b^2+c^2)}\geq$$

$$\geq 1+\frac{2(a^2+b^2+c^2)}{2(a^2+b^2+c^2)-(a^2+b^2+c^2)}=1+\frac{2(a^2+b^2+c^2)}{a^2+b^2+c^2}=1+2=3.$$

To prove this inequality, we are using first Titu's Lemma and then the inequality $ab+bc+ca\leq a^2+b^2+c^2$.

Conclusion

Thus, some methods of proving inequalities are shown. For each of them, precise schemes of reasoning are defined, using which one can prove the proposed inequality. In addition, convincing examples are given for each scheme. At the same time, this topic can be offered to students in specialized mathematical classes, in optional classes, in preparation for Olympiads, for creative research activities. It should also be noted that it is advisable for a mathematics teacher to select a system of tasks for independent work for each proof method. A positive effect can also be obtained from inequalities that can be proven in various ways.

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