EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6972

LINEAR DIOPHANTINE EQUATIONS AND METHODS FOR THEIR SOLUTION

Karimova Nargiza Mamurkhanovna
Military Academic Lyceum of Tashkent
'Temurbeklar maktabi' of the National Guard
of the Republic of Uzbekistan

Abstract

This article is addressed to students covering the topic of 'first degree Diophantine equations & their solutions'. In the article the main focus is on Diophantine equation solutions via geometric interpretation. The concern for Diophantine equations is clearly bound to human nature – its traces are found in preserved documents over thousands of years. Diophantine equations also give opportunity to solve algebraic problems using integers.

Keywords: cryptography, Diophantine equations, solution, algorithm, integers, selection, equality, unique solution

Introduction

Diophantine equations, which are integer equations, have several interesting applications in military security:

1. Resource optimization

Diophantine equations can be used for distribution of limited resources (e.g. for ammunition, machinery, troops) under specific conditions. For example, there may be a problem of how military units can be distributed for different purposes to maximise its efficiency.

2. Operation planning

When planning operations, it is important to consider integer limits e.g. the number of available means of transport or soldiers. Diophantine equations help find solutions, matching those limits.

3. Cryptography

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6972

Diophantine equations are used in some cryptographic algorithms for ensuring safety of data transmission. It can especially be useful to secure communication in the military environment.

4. Coding and decoding of messages

Methods based on Diophantine equations can be applied for creation and analysis of codes used for encryption of messages, which is crucial for confidentiality of military information.

5. Control systems and automation

"In the development of control systems for military technology, such as unmanned aerial vehicles (UAVs), Diophantine equations can assist in solving integer programming problems, for example, for optimizing trajectories and routes.

These applications demonstrate that Diophantine equations can be useful in various aspects of military defense, enhancing the effective use of resources and communication security."

Rule 1: If c is not divisible by d, then the equation $ax + by = c$ has no integer solutions. (GCD (a,b) = d).

Rule 2: To find a solution to the equation $ax + by = c$ when a and b are coprime, you first need to find a solution $(x_o;y_o)$ for the equation $ax + by = 1$; then Cx_o, Cy_o constitutes a solution for $ax + by = c$.

Solve the integer equation $5x - 8y = 19$ (1)

Solution

First method: Finding a particular solution by trial and writing the general solution. [2.c.11]

We know that if GCD $(a, b) = 1$, i.e., a and b are coprime, then equation (1) has integer solutions x and y. Since GCD (5, 8) = 1, we can find a particular solution by trial: $x_o = 7$; $y_o = 2$. Thus, the pair of numbers (7, 2) is a particular solution to equation (1).

This means the equality holds: $5 x 7 - 8 x 2 = 19 \ldots$ (2)

Question: How to express all other solutions given one solution?

Subtracting equation (2) from (1), we get: $5(x - 7) - 8(y - 2) = 0$.

From this, $x - 7 = \frac{8(y-2)}{5}$. It can be seen that (x - 7) will be an integer if and only if (y - 2) is divisible by 5, i.e., y - 2 = 5n, where n is any integer. Thus, $y = 2 + 5n$ and $x = 7 + 8n$, where n in {Z}.

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6972

Therefore, all integer solutions to the original equation can be expressed as:

$$\begin{cases} x = 7 + 8n, \\ y = 2 + 5n. \end{cases} \quad n \in Z.$$

Second Method: Solving the equation for one unknown. [1.c.13]

We solve this equation for the unknown with the smallest coefficient in absolute value. From $5x - 8y = 19 \Leftrightarrow 5x = 8y + 19 \Leftrightarrow x = \frac{8y+19}{5}$.

Calculating the remainders when dividing by 5: 0, 1, 2, 3, 4. We substitute these values for y:

If $y = 0$, then $x = \frac{8 \times 0+19}{5} = \frac{19}{5}$.

If $y = 1$, then $x = \frac{8 \times 1+19}{5} = \frac{27}{5}$.

If $y = 2$, then $x = \frac{8 \times 2+19}{5} = \frac{35}{5} = 7$.

If $y = 3$, then $x = \frac{8 \times 3+19}{5} = \frac{43}{5}$.

If $y = 4$, then $x = \frac{8 \times 4+19}{5} = \frac{51}{5}$.

Thus, a particular solution is the pair (7, 2). The general solution remains:

$$\begin{cases} x = 7 + 8n, \\ y = 2 + 5n. \end{cases} \quad n \in Z$$

Third Method: Universal method for finding a particular solution. [1.c.64]

We will use the Euclidean algorithm. We know that for any two natural numbers *a* and *b* such that GCD (a, b) = 1, there exist integers *x* and *y* such that $ax + by = 1$.

Plan:

1. First, solve the equation $5m - 8n = 1$ using the Euclidean algorithm.
2. Then find a particular solution to equation (1) using Rule 2.
3. Write the general solution for this equation (1).

1. Find the representation: $1 = 5m - 8n$. For this, we use the Euclidean algorithm:

$8 = 5 \times 1 + 3.$

$5 = 3 \times 1 + 2.$

$3 = 2 \times 1 + 1.$

From this equality, we express 1:

$1 = 3 - 2 \times 1 = 3 - (5 - 3 \times 1) \times 1 = 3 - 5 \times 1 + 3 \times 1 = 3 \times 2 - 5 \times 1 = (8 - 5 \times 1) \times 2 - 5 \times 1 =$

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6972

$= 8 \times 2 - 5 \times 2 - 5 \times 1 = 5 \times (-3) - 8 \times (-2)$.

Thus, m = -3, n = -2.

2. Particular solution for equation (1): $x_o = 19m$; $y_o = 19n$.

From this, we get: $x_o = 19 \times (-3) = -57$; $y_o = 19 \times (-2) = -38$.

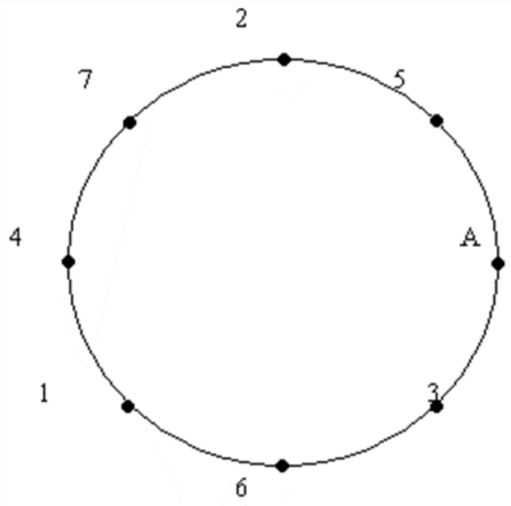
The pair (-57, -38) is a particular solution to (1).

3. General solution for equation (1): $\begin{cases} x = -57 + 8n, \\ y = -38 + 5n. \end{cases}$ $n \in Z$.

Fourth Method: Geometric. [3]

Plan:

1. Solve the equation $5x - 8y = 1$ geometrically.
2. Write the particular solution to equation (1).
3. Write the general solution for this equation (1).



We will sequentially set equal $\frac{5}{8}$ arcs on a circle, forming a regular octagon inscribed in the circle using 8 steps. After 5 full rotations, we reach a vertex adjacent to the starting point, having made 3 full rotations and passed the $\frac{1}{8}$ part of the circle, so $x \times \frac{5}{8} = y + \frac{1}{8}$.

Thus, $x_o = 5$, $y_o = 3$ is a particular solution for the equation $5x - 8y = 1$.

2. Particular solution for equation (1): $x_o = 19 \times 5 = 95$; $y_o = 19 \times 3 = 57$.

3. General solution for equation (1):

$\begin{cases} x = 95 + 8n, \\ y = 57 + 5n. \end{cases}$ $n \in Z$.

Conclusion

Diophantine equations remain one of the most exciting and significant areas in mathematics. From their historical roots in the works of Diophantus of Alexandria to modern breakthroughs and interdisciplinary research, these equations continue to arouse interest and stimulate intellectual curiosity.

Ultimately, the study of Diophantine equations continues to be an integral and inspiring part of the mathematical journey, offering endless opportunities for

EduVision: Journal of Innovations in Pedagogy and Educational Advancements
Volume 01, Issue 04, April, 2025
brightmindpublishing.com
ISSN (E): 3061-6972

exploration, discovery, and innovation, reflecting the richness and diversity of mathematical thinking.

References

1. E.P. Grinko, A.G. Golovach EDUCATIONAL AND METHODOLOGICAL MANUAL METHODS FOR SOLVING DIOPHANTINE EQUATIONS IN PREPARING SCHOOLCHILDREN FOR OLYMPIADS Brest BrSU named after A.S. Pushkin 2013
2. G. Falin, A. Falin. Linear Diophantine equations. M., Chistye Prudy Publishing House, 2008
3. https://urok.1sept.ru/articles/501260
4. https://poe.com/s/coVGmTDID6Gx4ZFWS3CS

.