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SOME PROBLEMS OF LINEAR PROGRAMMING WITH A LARGE NUMBER OF VARIABLES

Poshaxodjayeva Gulnora Djabborxanovna
Associate Professor of the Department of Higher and
Applied Mathematics Tashkent State University of Economics

Abstract
This scientific work considers some types of problems related to determining optimal plans for the aggregate material production of individual industries and enterprises using linear programming.

Keywords: Linear programming, matrix, costs, value, balanced production plan, final product, objective constraints, planned constraints, production volume, optimal production option.

Introduction

Some linear programming problems with a large number of variables. In most cases, the search for optimal plan options is associated with solving linear programming problems with a large number of variables.

A. Determining the optimal plan of material production. Let us assume that 1) the production of raw materials and supplies is divided into n industries; 2) the matrix B = [] (i, j = 1, 2, …, n) of material cost coefficients in the planning period is given; 3) the values of final products , , …. , (in value terms) are given for the period preceding the planning period.$b_{ij} x_1^{(0)} x_2^{(0)} x_n^{(0)}$

If the production volumes of individual industries are designated as , , …, , then the condition for the internal balance of the plan is a system of equations:$x_1 x_2 x_n$

$$b_{11}X_1 + b_{12}X_2 + \cdots + b_{1n}X_n + X_1 = X_1\ ,(1)$$

$$b_{21}X_1 + b_{22}X_2 + \cdots + b_{2n}X_n + X_2 = X_2\ , b_{n1}X_1 + b_{n2}X_2 + \cdots + b_{nn}X_n + X_n = X_n$$

Where are the planned final products of individual branches of production.$\mathrm{x_1 x_2 x_n}$

Different variants of the final social product correspond to different variants of production plans of individual industries. The optimal variant of the material production plan can be recognized as such a variant that, for example, ensures the

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receipt of a final product no less than in the period preceding the planned one, and the maximum volume of production in value terms. Consequently, the optimal variant of the plan will be the one that satisfies the conditions:

$$x_i = X_i - \sum_{j=1}^{n} b_{ij} X_j \geq X_i^{(0)} \ (i = 1, 2, \dots, n), (2)$$

$$\sum_{i=1}^{n} X_{i=\max} \ . (3)$$

The following factors are required for production:
1) objects of labor p,
2) means of labor m,
3) labor force s,
which are available in limited quantities.
When constructing an optimal version of a production plan, it is necessary to determine:
1) consumption rates of production factors available in limited quantities;
2) the size of stocks of production factors; 3) the lower limits of the volumes of final products. Let us take the maximum of the final social product as the criterion for optimal production; let the matrix

$$\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ m_{11} & m_{12} & \dots & m_{1n} \\ S_{11} & S_{12} & \dots & S_{1n} \end{bmatrix}$$

will be the matrix of production factor consumption rates (per unit of final product). Let us also assume that the sizes of production factor stocks are P, M, and S, respectively.
Therefore, the optimal version of the material production plan will be the one that satisfies the following conditions:

$$x_i = X_i - \sum_{j=1}^{n} b_{ij} X_j \geq x_i^{(0)} \ (i = 1, 2, \dots, n), (2)$$

$$p_{11}x_1 + p_{12}x_2 + \cdots + p_{1n}x_n \leq P \ , (4)\ (5)\ (6) m_{11}x_1 + m_{12}x_2 + \cdots + m_{1n}x_n \leq M \ , s_{11}x_1 + s_{12}x_2 + \cdots + s_{1n}x_n \leq S \ ,$$

$$. (7) \sum_{i=1}^{n} x_{i=\max}$$

Conditions (4), (5), (6) mean that the expenditure cannot exceed the resources of these factors. The optimal variant of the material production plan can also be determined in another way. Let us assume that we know:

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1) the matrix of material costs
$B = [b_{ij}]$ $(i, j = 1, 2, \dots, n)$, 2) the cost-of-living labor per unit of output in value terms, 3) maximum employment in material production, 4) specified final products. As an optimal draft plan, we can adopt a variant in which the final product of the k-th industry is maximum, the final products of other industries are not less than the planned levels, and employment does not exceed the value of Z. Consequently, the optimal structure of production in individual industries will be one that satisfies the conditions:

$$z_j (j = 1, 2, \dots, n) z x_i^{(0)} (i = 1, 2, \dots, n, i \neq k).$$

(8) (9) (10) (11)

$$X_i - \sum_{j=1}^{n} b_{ij} X_j \geq X_i^{(0)} \ (i = 1, 2, \dots, n), \sum_{j=1}^{n} z_j X_j \leq Z, X_j \geq 0 \ (j = 1, 2, \dots, n), x_k$$

$$= X_k - \sum_{j=1}^{n} b_{kj} X_j = \max.$$

It is also possible to define other criteria for the optimality of the production plan. Let us assume that material production is divided into n branches, and that the inter-branch connections are determined by the matrix

$$B = [b_{ij}] \ (i, j = 1, 2, \dots, n).$$

Let us further assume that is the amount of labor required to produce a unit of output in the j-th industry, and that employment not related to production is . If the value of output in the j-th industry is denoted by , then determines the labor costs required to carry out production in the j-th industry, and the total social labor costs required to fulfill the production plan are:$z_j z_0 X_j z_j X_j$

$$\sum_{j=1}^{n} z_j X_j.$$

If the labor force resources are , then the equation must be satisfiedz_0

$$\sum_{j=1}^{n} z_j X_j + z_0 \leq Z_0.$$

In addition, there are balance equations for production:

$$X_i = \sum_{j=1}^{n} b_{ij} X_j \geq x_i \ (i = 1, 2, \dots, n),$$

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where is the final product of the i-th industry.x_i
The optimal variant of the production plan can be considered to be the one that ensures the receipt of the corresponding final products.

$$x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}$$

with minimal expenditure of social labor. Thus, it is necessary to determine X so that

$$Z = \sum_{j=1}^{n} z_j X_j = \min$$

under conditions: (13) (14)$X_i - \sum_{j=1}^{n} b_{ij} X_j = x_i^{(0)}\ (i = 1, 2, \dots, n), X_j \geq 0, x_i^{(0)} \geq 0\ (i, j = 1, 2, \dots, n),$

$$\sum_{j=1}^{n} x_j X_j + z_0 \leq Z_{0.}\ (15)$$

Inequalities in optimal planning problems represent certain restrictions on the choice of a plan variant. These restrictions can be divided into two groups:
1) objective constraints, 2) planned constraints. Objective constraints do not depend on decisions related to the plan, for example, on the state and structure of fixed assets at the beginning of the planning period, the size of supply and demand in external markets, existing production technology, labor resources available at the beginning of the period, etc. Planned constraints are established on the basis of certain socio-economic decisions. These may include such constraints as the necessary size of collective consumption, the volume of non-production capital investments, employment in material production, planned foreign exchange balances in circulation with individual external markets, planned levels of wages and other incomes of the population.
B. Definition of the plan of the production sector. Let the production sector be represented by an association subordinate to which are n industrial enterprises, each of which produces products 1, 2, …, m.
A production plan for individual products in quantities is established for the association. The planned production capacities of the enterprises (in working hours) are respectively, and the production costs per unit of product in the corresponding enterprise are the optimal plan will be one in which the production

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costs of the planned products of the association will be minimal. Let us designate the production volume of the i-th product in the planning period at the j-th enterprise as . The following conditions must be met: $A_1, A_2, \dots, A_m$ $T_1, T_2, \dots, T_n c_{ij} (i = 1, 2, \dots, m;\ j = 1, 2, \dots, n)$.

$$x_{ij} (i = 1, 2, \dots, m;\ j = 1, 2, \dots, n)$$

$$x_{11} + x_{12} + \cdots + x_{1n} = A_1,$$

$$x_{21} + x_{22} + \cdots + x_{2n} = A_2, \dots\dots\dots (1) x_{m1} + x_{m2} + \cdots + x_{mn} = A_m,$$

(some of the variables may be equal to zero). The second condition that the variables must fulfill is related to the fact that the volume of production at each enterprise cannot exceed its production capacity. Therefore, the following conditions must be met:x_{ij} x_{ij}

$$t_{11}x_{11} + t_{21}x_{21} + \cdots + t_{m1}x_{m1} \le T_1,$$

Obviously, the following conditions must also be met:

$$t_{12}x_{12} + t_{22}x_{22} + \cdots + t_{m2}x_{m2}$$

$$\le T_2, \dots\dots\dots\dots (2) t_{1n}x_{1n} + t_{2n}x_{2n} + \cdots + t_{mn}x_{mn} \le T_n.$$

$x_{ij} \ge 0$ $(i = 1, 2, \dots, m;\ j = 1, 2, \dots, n)$. (3)

Since the volume of production in the industry should be distributed among enterprises in such a way that the total production costs are minimal, the following condition must be met:

$$C = c_{11}x_{11} + c_{12}x_{12} + \cdots + c_{1n}x_{1n} +$$

$$(4) + c_{21}x_{21} + c_{22}x_{22} + \cdots + c_{2n}x_{2n} +$$

$$+ c_{m1}x_{m1} + c_{m2}x_{m2} + \cdots + c_{mn}x_{mn} = \min.$$

The solution to the problem comes down to finding such values of the unknowns that they satisfy conditions (1), (2), (3) and so that expression (4) takes on a minimum value. $x_{ij} (i = 1, 2, \dots, m;\ j = 1, 2, \dots, n)$

In the case under consideration, the optimality criterion is production costs. Another optimality criterion can also be adopted, for example, maximum use of production capacities of enterprises. Then the optimal variant of the unification plan will be the one for which conditions (1), (2), (3) and

$$\sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij}\, x_{ij} = \max. \ (5)$$

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Example. As an example, let us consider the optimal plan for the placement of agricultural production. Let the area of arable land be divided into two zones by yield per hectare. The yield of four types of grain and potatoes, as well as labor costs (in working hours) are given in the following table:

	Grain by zone		Potatoes by zone	
	I	II	I	II
Productivity (c).	18	20	110	70
Costs (in working hours) . . .	0.8	1	11	14

The resources of production factors are as follows:

	Zone	
	I	II
Agricultural land (thousand hectares) . . . .Working days (thousan) . . .	650 1000	550 1200

actors of production by zones	Unit of measurement	Zone I		Zone II		Resources of production factors (thousand)
		Cereals	Potato	Cereals	Potato	
Agricultural land in zone I . . . **Working days in zone I . .** **Agricultural land in zone II . . .** **Working days in zone II.**	ha. unit ha. unit	0.04 0.04	0.02 0.2	0.0725 0.12	0.01 0.17	650 1000 550 1200

The price of grain is set at 220 zlotys per centner, and potatoes at 130 zlotys per centner. According to the plan, grain production should be no less than 15 million

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centners, and potatoes no less than 7 million centners. From these data it follows that the costs of production factors per centner of output are the values given in the table:

The planned production volumes of grain and potatoes in zone I are denoted by x1, x2, and in zone II - x3 and x4. We have the following restrictions:

$$0.04x1 + 0.02x2 \leq 650{,}000 \quad (6)$$
$$0.04x1 + 0.2x2 \leq 1{,}000{,}000 \quad (7)$$
$$0.0725x3 + 0.01x4 \leq 550{,}000 \quad (8)$$
$$0.12x3 + 0.17x4 \leq 1{,}200{,}000 \quad (9)$$
$$x1 + x3 \geq 15{,}000{,}000 \quad (10)$$
$$x2 + x4 \geq 7{,}000{,}000 \quad (11)$$

If the maximum cost of agricultural production is taken as the optimality criterion, then the optimal variant of the agricultural production plan will be the one in which the unknowns x1, x2, x3 and x4 satisfy conditions (6) - (11) and in which the expression for the unknowns is

$$(x1 + x3)\,220 + (x2 + x4)\,130$$

takes on its maximum value.

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